

# MATHEMATICS IN PRE-SERVICE TEACHER EDUCATION AND THE QUALITY OF LEARNING: AN EXPERIENCE WITH PAPER PLANES, SMARTPHONES AND GEOGEBRA

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*This poster presents an instance of mathematics modelling by students on a geometry activity in a pre-service teacher training course analysed through a model designed to access the mathematical thought and the quality of student leaning outcomes. This analytical model, supported by the SOLO taxonomy, uses Activity Theory as a contextual framework that integrates the different relations, namely advanced mathematical thinking concepts like procept and proceptual divide. Results allowed us to see three different pathways taken by the students to solve the same problem.*

*Keywords: Advanced mathematical thinking, geometry, proceptual divide, quality of learning, SOLO taxonomy.*

## INTRODUCTION

Portfolio assessment brings an open evaluation method into the mathematical classroom and allows the mathematical abilities of the students to grow. In this study, students use paper planes, smartphones and dynamic geometry software (geogebra) to find the equation that describes the flight path of the paper planes and were given two classes to solve them and to explain in detail their solution process. This solution process involves brainstorming sessions centred on the best solution, and the detailed explanation necessarily involved in self-regulated learning processes. This teaching method aims to extend the mathematical knowledge of future teachers, involving them in activities more open and less structured than the traditional ones.

The data presented here were chosen because it highlights three different path and took different approaches to the same problem. Data was studied using the analytical model that highlights these differences and integrates SOLO taxonomy (Biggs & Collis, 1982) with the advanced mathematical thinking theories and concepts of Tall (2002) alongside the conceptualization of the proceptual divide (Gray & Tall, 1994), and activity theory (Engeström, 2001) as a contextual structure. The SOLO taxonomy allows us to identify five progressive levels of understanding from the *prestructural* (lowest level), through the *unistructural*, the *multistructural*, the *relational* to the *extended abstract* (highest level).

The task statement asks to find the equation that describes the flight path of a paper plane, using smartphones and geogebra, writing a step-by-step report of the activity. In the first class students started by folding the paper planes (exploring somehow the art of origami), all the instructions were given using geometry concepts like area, midpoint, segment bisector and so on. With the planes built, the group moved outside the classroom and, using the cameras of their smartphones, photographed the various moments of the flight of the planes since its launch, the flight itself and the landing. Photographs sequences were drawn according to contemplate various angles, forces and aircraft launch positions and capture the movement of the same in flight and subsequent landings.

After the images were collected, they were recorded on the computers and using geogebra, the flight sequence of the various planes was simulated using a Cartesian axis and the plane's nozzle as

a point of reference for marking points. With the complete sequence of the flight scanned in points in the software the described curve of the airplane was worked in order to find one, or more equations of the function described by the airplane. This trajectory (usually a parabola - represented by a second degree equation) can be shown and students can control the parameters of the same, appropriating algebraic concepts. In this case, factors like the idea of air resistance and elevation were not contemplated, which would allow other types of explorations (which were not the objective of this task).

## **FINAL REMARKS**

This form of mathematical modelling becomes attractive and allows students to visualize mathematics to take shape as a real activity, evading the notion of common sense that mathematical concepts do not represent reality. Geogebra was introduced in the task, and students learned their handling by trial-and-error. To finalize the exploration task, hypothetical situations were created with the software that served as an exercise to study the characteristics of a quadratic function and the parameters of its variables. Experience had good adhesion by the students and were in some expectant form on the final results, some of the comments involved the strangeness of a math class have to go out of the classroom (which, in most cases it was an absolute novelty).

The results of the analysis identify three levels of response: a first level, where the construction of the function fails to recognize the graphical movement of the various parameters of the quadratic function, identify the vertex, place the parabola in the correct place, but show difficulties in adjusting the aperture of the parabola (roots of the quadratic equation) to the curve of the airplane were classified as of the prestructural level; a second level where, when debating with the same problems of the previous level, they use the calculation (with paper-and-pencil or with graphing calculator - even having the software of dynamic geometry available) to adjust the parameters of the function and were classified as multistructural level, possibly relational and; a third level where using the functions of the selectors adjusted the parabola to the plane curve, arriving with some ease to the quadratic equation classified as relational level, possibly extended abstract.

Given the constraints, such as the lack of knowledge of the software and the lack of habituation to tasks of this kind, allowed the students to have a different view of what can be an attractive math class for the students, which is evidenced with some of the conclusions of the students themselves:

Student A: We can also verify that math can be fun and does not have to be just worked indoors and with the use of new technologies we can catch attention and perform a certain type of exercises easier (student report).

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