

THE TRANSPOSITION OF COUNTING SITUATIONS IN A VIRTUAL ENVIRONMENT

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The mathematics education research is increasingly focused on different didactical hypotheses for constructing teaching and learning situations involving the decimal principle of the numeration system. One of these situations is, for example, counting a big collection of objects through the tangible manipulation. In this paper we introduce the simulating device, “Simbûchettes”, for analysing its potential concerning this situation with respect to the tangible material. In particular, we will show that “Simbûchettes” preserves all of the techniques we identified in the tangible world and it allows to mobilise other techniques strongly grounding on the decimal principle of the numeration system that we rarely observed with the tangible material.

Keywords: TAD, simulating device design, tangible manipulation, decimal number system

INTRODUCTION

Learning how decimal number system works has an important role in understanding several areas of mathematics: the calculus, the conversion of units of measurement, the decimal numbers, etc. The decimal number system is the product of the articulation between two different principles: the decimal and the positional one (Serfati, 2005). The “positional principle” allows associating a rank within a string of digits to each numbers unit (ones, tens, hundreds, thousands...). In other words, the position of a digit in the number determines its value. The “decimal principle” explains the relations between different numbers units in a number: each unit is equal to ten units of the next lower rank (for example, 1 ten=10 ones, 1 hundred=10 tens= 100 ones; 1 thousand=10 hundreds= 100 tens=1000 ones ...).

THEORETICAL PERSPECTIVES

According to Tempier (2010), the decimal principle is considered as a source of learning difficulties. However, this aspect is necessary for understanding the numeration system. This principle can be taken into account, for example, through activities involving grouping by tens and exchanges. These rules state that 10 elements of a numbers unit can be grouped and exchanged with one element of the numbers unit of the next bigger rank. Moreover, one element of a numbers unit can be ungrouped and exchanged with 10 elements of the numbers unit of the next lower rank.

Difficulties related to teaching the numeration system at primary school

Research carried out by Bednarz and Janvier (1984) has shown students’ difficulties related to understanding the decimal principle of the numeration system:

- *“difficulty in seeing groups of tens and their role in the canonical form of written-numbers, despite the important place that this canonical form takes in teaching”;*

1. *“difficulty in seeing the relevance of these groups of tens”;*
2. *“difficulty in working with these groups of tens, in terms of constructing and deconstructing*

- them*”;
3. “*difficulty in working with two different groups of tens at the same time*”;
 4. “*difficulty in the interpretation of the calculus procedures in relation to the mathematical operations (additions, subtractions, multiplications, divisions), in terms of groups of tens that leads to classic errors on the operations*” (Bednarz and Janvier, 1984).

The analysis of pupils’ errors carried out by Parouty (2005) reinforces the fourth of these difficulties on the relation between the numeration system and the calculus. The activities observation proposed in the mathematics classroom explains why students have these type of difficulties. Actually, these activities mostly concern the positional principle of the numeration system. This is why pupils’ learning is principally based on this aspect. Bednarz and Janvier (1984) make the same observation regarding the activities choice. For example, “the number representation appears according to the alignment related to the canonical form of the written-number”. 26 years later, Tempier (2010) finds the same difficulties in students. For these reasons, a lot of researches focuses on how to take into account the decimal principle of the numeration system in the current teaching. In particular, different research studies have pointed out several working hypotheses on which constructing didactical situations highlighting the decimal principle (Tempier, 2010; Chaachoua 2016).

Didactical hypotheses

In this paper, we will consider three of these didactical hypotheses developed in Chaachoua (2016).

The relations among numbers units

According to Chambris (2008), it is very important to consider the different relations among the numbers units for mobilising the decimal principle, favouring the conversion tasks, for example converting 23 hundreds into tens. That is why our first working hypothesis is:

(HT1) “*The relations among numbers units*”: *to work on the numeration system, we have to consider tasks that mobilise the relations among numbers units.*

Big numbers

Big numbers allow us to explicitly work on numeration system and, particularly, on the decimal principle. Actually, the introduction of a new numbers unit produces different relations among numbers units. This way, the repetition of grouping of tens and the exchanges allow us to better understand the decimal principle of the numeration system. Hence, our second working hypothesis is the following:

(HT2) “*Big numbers*”: *The introduction of big numbers increases and reinforces the understanding of numeration system and, particularly, its decimal principle.*

The objects’ manipulation

For teaching numbers and the numeration system, the objects’ manipulation constitutes a very important phase in sense-making.

According to Raoul-Bellanger and Bellanger (2010), the manipulation in mathematics allows pupils

to construct a mental image and to improve their abstraction capacity (iconic or symbolic system). This becomes really true for pupils with learning difficulties where the manipulation could be used in the remediation phase. Hence, our third working hypothesis is:

(HT3) “The objects’ manipulation”: the objects’ manipulation is important for practising rules concerning grouping of tens and exchanges for giving sense to the decimal principle of the numeration system.

Drawing on these different didactical hypotheses, we will focus on the type of task “Counting a big collection of objects”, through which we can take into account the decimal principle after having chosen a relevant collection configuration.

The scientific challenges

In general, within the tangible manipulation activities, the time for accomplishing some actions (i.e., grouping of tens) increases when the collection size is big. For this reason, for discouraging the employing of some techniques not adapted to fit with big numbers, it is necessary to repeat tasks and the manipulation becomes time consuming.

Monitoring pupils individually during their manipulation actions is a difficult task for the teacher. For this reason, the implementation of teaching and learning situation based on this manipulation encounters three obstacles: (1) the manipulation of big collections demands a lot of time, (2) the tangible objects don’t produce relevant retro-actions with respect to pupils’ learning and (3) the teacher cannot observe different pupils at the same time. The last point will not be discussed in this paper, but it is the research theme of the Ph.D. thesis of Brassat (2016).

Our research question is two-fold: *does this technological device preserve all of the characteristics of the tangible manipulation and how this simulating device can overtake the challenges (1) and (2) related to the type of task “Counting a big collection” based on the manipulation of tangibles objects?*

In this paper, we will focus on the first part of our research question.

THE SIMULATION DEVICE “SIMBÛCHETTES”

With respect to the challenges presented above, a research project is carrying out by the MeTAH team of the University of Grenoble Alpes and it concerns the design of a simulating device “Simbûchettes” and of an orchestration device for monitoring all of the students (Wang et al., 2017). In this paper, we will focus on the simulating device “Simbûchettes”. Our hypothesis is that this device can take into account the decimal principle of the numeration system, according to the previous part. In the frame of this project a simulation on a tablet (Fig. 1) has been developed. It allows us to manipulate virtual objects, to move small sticks, to put them into boxes, to group and ungroup them, to duplicate them, etc. All of the actions made on the tablet can be recorded. The treatment of these actions constitutes a retroaction for the pupils and they can also inform the teacher. The touch screen interface conception, the variables choice and the treatment of the actions are based on the didactical computing model T4TEL (Chaachoua and Bessot, 2016). This theoretical framework refers to the Anthropological Theory of Didactic (ATD) (Chevallard, 1992, 1998, 1999) and in particular to the praxeological approach.

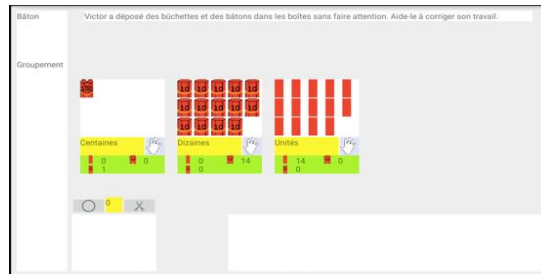


Fig 1. Example of the interface of the simulating device

This device gives us the possibility of choosing and defining the parameters which allow us to create different didactical situations. These parameters can concern the displaying of the constitutive elements of the interface (boxes, duplication zone, action buttons, etc.), the elements available for pupils and the actions that are authorised or not. We can configure the device forbidding some specific type of action. For example, we can forbid the introduction of a tenth element after having already put in a box 9 small sticks or 9 packs of small sticks. The device “Simbûchettes” gives the possibility to easily make and unmake groups of ten. It also gives the opportunity of producing exchanges among different numbers units focusing on their relations. Moreover, we can manage collections with a big number of elements. Actually, in terms of the time and equipment management, it allows us to make repetitions: this is an important condition to avoid costly techniques. This makes the teacher able to know all of the actions of pupils during the manipulation without coaching them one by one.

METHODOLOGY

This research involves 30 pupils in two third grade classes of a primary school of Grenoble. 7 of them have worked with the tangible material and the others 23 with the device “SimBûchettes” on the same activities concerning the type of task “Counting a collection”, in according to the didactical hypotheses explained in the theoretical part of the paper. This type of task has been studied in Chaachoua (2016) in which the author has developed an epistemological model of reference according to the theoretical framework T4TEL. We have relied on this study for conceiving our activities. In particular, we have focused on three different problems and many different activities that we resume in the table below. The question was always the same: “How many sticks?”

Problem	Example of an activity of the experimentation
1. Counting a collection “in bulk”	80 sticks “in bulk”
2. Counting a homogeneous collection	9 tens of sticks, 12 tens of sticks, 67 tens of sticks
3. Counting a completely ordered collection	2 hundreds of sticks, 23 tens of sticks, 15 sticks

Table 1. General description of the different tasks proposed to the pupils with examples

Pupils’ actions were video recorded during the activities with the camera facing their hands and the material on the table in the case of the tangible experiment or the tablet in the other one. All voice

and hands movement during the activity were recorded. The videos were transcribed for data analysis.

In the following, we will describe pupils' techniques we have observed for solving the different tasks listed above. Concerning the structure of the analysis, in the first part, we will show that in the virtual environment, we have observed the same techniques appeared in the tangible one, even if the implementation of a same technique is deeply different in the two cases. In the second part, we will go further showing how "Simbûchettes" produces other techniques that strongly mobilise the decimal principle.

DATA ANALYSIS

In the first part of the data analysis, for each problem, we are going to present the different techniques we have identified during the teaching experiments with "Simbûchettes".

Problem 1:

- **Technique 1.1:** Grouping by tens, counting by tens (or by 20 or by 30).
- **Technique 1.2:** Grouping by tens, counting by numbers units, converting numbers units to ones.
- **Technique 1.3:** Counting by n , where n is 1, 2, 3...

Problem 2:

- **Technique 2.1:** Grouping by tens, counting by numbers units, converting numbers units to ones.
- **Technique 2.2:** Grouping and counting by X , where X is a power of ten (1, 10, 100...).
- **Technique 2.3:** Counting by n , where n is 1, 2, 3...








Problem 3:

- **Technique 3.1:** Counting separately each rank by number units, converting the number units to ones and adding them.
- **Technique 3.2:** Counting separately each rank by number units, converting the number units to ones and thinking to the configuration of the written-number.

As shown above, the simulating device doesn't avoid the techniques related to the tangible manipulation presented in Chaachoua (2016). This point answers to the first part of our research question: from an ecological point of view, the simulating device preserves all of the characteristics of the tangible manipulation. Moreover, we can highlight that the simulating device enhances the variety of techniques: for example, we have observed that the technique of "grouping and counting by hundreds" is present in the virtual manipulation, while it doesn't appear in the tangible one. Moreover, in general, all of the pupils employed the technics of "grouping by tens" in the virtual environment, while in the tangible one they used very often other technics without making groupings. These observations lead us to hypothesize that with "SimBûchettes" it is more natural for pupils to make grouping of tens or of hundreds with respect to the tangible material.

In the second part of the data analysis, we will show the mathematical activity of two pupils (S1 and S2 in the following) who employ different technics for the same activities in the two environments.

Three months have passed between the tangible teaching experiment and the virtual one. During the experimentations, before starting the activities, the researchers (R in the following) gives some preliminary information about the material pupils had at disposal. For the tangible material, the researchers said that each package of sticks contains exactly 10 sticks and each package has the same cardinality and that there were some elastics on the table that pupils can use if they wish. For the virtual environment, the researchers explains the different elements constituting the interface of “SimBûchettes”.

S1	Activity: 12 tens of sticks		Activity: 67 tens of sticks	
	Tangible	“SimBûchettes”	Tangible	“SimBûchettes”
	<p>First of all, S1 counts the number of sticks in a package (see Fig.1)</p>  <p>Fig. 1: S1 counts the sticks in one package</p> <p>After, S1 mentally counts the number of packages, taking in his hands the packages one by one (see Fig.2).</p>  <p>Fig. 2 S1 counts the packages</p>	<p>S1 immediately constructs one grouping of tens for making the hundreds (see Fig.3)</p>  <p>Fig. 3: On the table zone there are the package of hundreds and two packages of tens</p>	<p>S1 makes groupings of two tens without using the elastic and he puts them on one of the corners of the table (see Fig.4).</p>  <p>Fig. 4: S1 makes the groupings of two tens without using the elastic</p> <p>Then, he decides to take an elastic for encircling the packages of two tens he made (see Fig. 5).</p>  <p>Fig. 5: S1 takes an elastic for encircling the grouping of two tens he made</p>	<p>S1 makes immediately groupings of hundreds (see Fig. 6 and Fig. 7)</p>  <p>Fig. 6 S1 is moving the packages of tens in the construction zone of the interface</p>  <p>Fig. 7 At the end, S1 has on the table 6 groupings of hundreds and 7 groupings of tens</p>


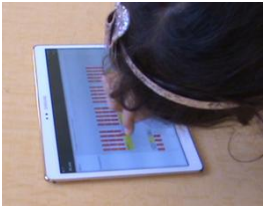

Transcription	<p>S1: 120 R: How did you do? S1: 10 by 10 R: 10, 20, 30, Isn't it? S1: yes</p>	<p>R: How did you do? S1: I have found that there were 12 packages of tens, I have put 10 packages of tens here (<i>pointing to the zone of construction of grouping of tens</i>). Then, there were still two packages, so it is 120.</p>	<p>S1: 680 R: How did you do? S1: I started to count 20 by 20, but then I finished counting 10 by 10.</p>	<p>S1: 670 R: Can you count aloud? S1: 100, 200, 300, 400, 500, 600, 610, 620, 630, 640, 650, 660, 670 (he points the packages while he counts).</p>
Analysis	<p>S1 counts the sticks 10 by 10 (without making grouping of hundreds), after having verified that in one package there were 10 sticks.</p>	<p>In this case, S1 counts the number of packages and he makes a grouping of hundreds. Then, he makes 100 plus 20.</p>	<p>S1 started to make the grouping of two tens for counting 20 by 20, probably because he wanted to save time. But, then, he continued to count 10 by 10, probably because it was simpler for him to count 10 by 10 with respect to 20 by 20. At the end, he made an error in the calculus.</p>	<p>S1 makes groupings of hundreds and he counts 100 by 100.</p>
S2	Activity: 80 sticks "in bulk"		Activity: 67 tens of sticks	
	Tangible	"SimBûchettes"	Tangible	"SimBûchettes"
	<p>S2 counts without moving the sticks (see the sequence of figures below), even if many times the researcher said to her that she could move them.</p> 	<p>S2 makes groupings of tens in the construction zone of the interface (see Fig. 8) and, then, she counts the packages she made (see Fig. 9).</p> 	<p>S2 counts without moving the sticks (see the sequence of figures below), even if many times the researcher said to her that she could move them.</p> 	<p>S2 makes groupings of hundreds in the construction zone of the interface (see Fig. 10) until she makes 6 packages of hundreds. After she tries to make another grouping of hundreds but she becomes aware of the fact that there are 7 grouping of tens instead of 10. She decides to leave the 7 packages of</p>



Fig. 8 S2 moves the sticks in the grouping zone for making packages of 10

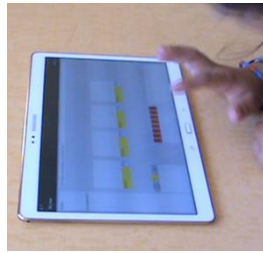


Fig. 9 She counts the number of packages she has



10 in the construction zone (see Fig. 11) and she counts.



Fig. 10 She moves the last 7 packages of 10 she had for trying to make another package of 100, but she becomes aware of the fact that the number of packages is not sufficient for making 100



Fig. 11 She left the 7 packages in the grouping zone and she begins to count.

Transcription	<p>R: Can you count this way? S2: uhm R: How many ? N: 74 R: 74, ok, how did you do? S2: <i>silence</i> R: Don't you know? Did you try to count all of the sticks? S2: yes</p>	<p>S2: I'm done R: How many sticks are there? S2: 80 R: Can you explain me how did you do? S2: I organised the sticks by groupings of tens. R: And how did you find the number 80? S2: I did 10 plus 10, 20, plus 10, 30 plus 10, 40 [...] 80.</p>	<p>S2: 500 R: 500? S2: yes</p>	<p>S2: I'm done R: How many sticks are there? S2: 670 R: Can you explain me how did you do? S2: I did 100 plus 100 plus 100 plus 100 plus 100 plus 10 plus 10 plus 10 and in the end I added all together.</p>
Analysis	<p>S2 counts the sticks one by one without moving them, just pointing them while she counted and changing the position of her body for seeing the hidden sticks. At the end, she found 74 sticks that it is not too far from the correct answer.</p>	<p>In this case, S2 is more self-confident of her result with respect to the other case: in fact, she called the researcher saying "I'm done". She makes groupings of ten and then she counts 10 by 10.</p>	<p>S2 probably counts 10 by 10 without moving the packages.</p>	<p>S2, after making grouping of hundreds, makes an addition, founding the correct answer.</p>

DISCUSSION AND CONCLUSION

As we discussed in the data analysis, the technological device allows to produce the same techniques identifying with the tangible material and it enhances others techniques (i.e., making grouping of hundred) deeply linked to the decimal principle of the numeration system that we have rarely observed in experimentations with the tangible material. From an ergonomic point of view, probably, it is not easy for pupils to make, for example, grouping of hundred with tangible sticks. Moreover, as we shown above, even a same pupil who immediately makes grouping of hundreds with "Simbûchettes", three months earlier, for the same activity, he preferred to count 10 by 10 without making packages of 100. This element gives us some first information about the potential of "Simbûchettes" with respect to the tangible material. In the future research, we will try to investigate more deeply this potential answering to the second part of the research question "*how this simulating device can overtake the challenges (1) and (2) related to the type of task "Counting a big collection" based on the manipulation of tangibles objects?*" We will study if "Simbûchettes"

could reduce the time of manipulation with respect to what happens in the tangible environment. Anyway, we can already state that, with “Simbûchettes”, the teacher reduces her time regarding the preparation of the different configurations of collections. Moreover, even if the time will not be different, we could investigate the different implications that the two environments have on students’ learning and how the role of the retro-actions offered by the simulation device could enhance them.

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