An interactive book on axial symmetry and the synergic use with paper and pin

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This work presents results from a teaching experiment concerning the construction-conceptualization of axial symmetry at Primary School through an interactive book, developed in a Dynamic Geometry Environment (DGE), which embeds a set of tasks to be accomplished with selected DGE tools. The tasks are part of a teaching sequence, framed by the Theory of Semiotic Mediation (TSM), whose main characteristic is the synergic use of a "duo of artefact". The duo is made up of a digital artefact - the interactive book - and a manipulative artefact, constituted by paper and pin. Herein, we describe the design of the interactive book and we show how a cognitive synergy arises from its use combined with the use of the manipulative artefact within the sequence, thus leading to the conceptualization of mathematical meanings.

Keywords: Synergy between artefacts, Duo of artefacts, Digital Artefacts, Dynamic Geometry Environments, Theory of Semiotic Mediation

INTRODUCTION

Nowadays scholars generally agree that the use of tools, being manipulative or digital artefacts, can have potential to enhance mathematical understanding (Monaghan et al., 2016). In particular, many researchers have investigated on the use of Dynamic Geometry Environments (DGEs) in the mathematical teaching and learning processes. Leung (2008), for instance, underlies that a DGE has the ability to visually make explicit the implicit dynamism of "think about" mathematical concepts. The dragging function in DGE, indeed, allows to perceive patterns of variation and to discover invariant properties, thus playing a key role in the construction of mathematical meanings. However, different epistemological approaches to mathematical learning have different implications on designing tool-based teaching and learning activities (Leung and Bolite-Frant, 2015). For example, tools can be seen as mediators for mathematical discourse (Sfard, 2008) or as psychological tools in the context of social and cultural interaction, developed through the zone of proximal development and internalization processes (Vygotsky, 1978).

This work is part of a research project developed in a Vygotskian perspective and, more precisely, under the overarch of the Theory of Semiotic Mediation (TSM) (Bartolini Bussi and Mariotti, 2008), in which artefacts can be seen as tools of semiotic mediation. In order to design a teaching sequence, aimed at fostering the construction/conceptualization of axial symmetry at Primary School, we have considered a "duo of artefacts" (Maschietto and Soury-Lavregne, 2013), composed by a manipulative artefact and a digital one. The choice of the artefacts has been done with the aim to develop a synergy between their use, whereby the potential of the activities with the artefacts would be enhanced.

In this paper we describe, in particular, the design of the digital artefact, an Interactive Book (IB) developed in a DGE. The IB is presented focusing on the semiotic potential of its use, according to the TSM, within the designed sequence, in which it is combined with the use of the manipulative artefact.

Moreover, we present and discuss outcomes from the experimentation of the sequence, aiming to answer to the following research question: can the synergic use of our duo of artefacts develop a cognitive synergy fostering the conceptualization of axial symmetry?

THEORETICAL FRAMEWORK

The Theory of Semiotic Mediation, developed by Bartolini Bussi and Mariotti (2008), deals with the complex system of semiotic relations among: the artefact, the task, the mathematical knowledge that is the object of the activity, and the teaching/learning processes that take place in the class.

According to it, in semiotic activities various signs are produced: the "artefact signs", that often have a highly subjective nature and are linked to the learner's specific experience with the artefact and the task to be carried out; the "mathematical signs", in other words the knowledge of mathematics to which the "artefact signs" must evolve; and finally the "pivot signs", that illustrate the evolution between artefact signs and mathematical signs, through the linked meanings.

The role of the teacher is to foster, through Mathematical Discussions (Bartolini Bussi, 1998), the shared construction of mathematical signs, guiding the evolution of personal meanings toward mathematical meanings. In the design of our teaching sequence, we followed the general scheme of successive "didactic cycles", which organize the coordination between activities with the artefact and semiotic activities, finalized to make the expected evolution of signs occur.

Moreover, in the design process of the teaching activities we focused upon the "semiotic potential" of the artefact, that is the basis underlying, on the one hand, the design of the teaching activities and, on the other, the analyses of both the actions and production of signs and the evolution of meanings.

To complete the description of the theoretical framework of this research we need to refer to the notion of "duo of artefacts". Maschietto and Soury-Lavregne (2013) have designed a digital artefact corresponding to a given physical artefact in order to investigate if such a "duo of artefacts", can enlarge and improve the learning experience of the students. In our study, as in their duo, the two artefacts must have some common characteristics, enabling transfer and reinvestment from one to the other. For this reason, whilst our digital artefact is not a digital counterpart of the manipulative artefact, we do use this notion in our work as well.

RESEARCH METHODOLOGY

Following the teaching experiment methodology (Steffe and Thompson, 2000), a teaching sequence has been designed in conformity with the chosen theoretical framework and the formulated hypothesis. It constructs the environment where the data, on which to analyse the results of the experiment, are collected. The sequence is framed on the TSM taking into account a theoretical reflection on the meaning of axial symmetry, with its definition and its properties, and an a priori analysis of the semiotic potential of the artefacts. It has been implemented with fourth grade students in a pilot study, involving two groups of four pupils, and in a further study, involving a whole class of twenty pupils. The teaching experiments were videotaped and conversations were transcribed, that also took into account the specific actions taken with the artefacts. The videotapes and transcriptions were then used to analyse the teaching experiments.

Analysis of the pilot study results, not only showed that the sequence contributed to the emergence and evolution of signs – in line with what expected by the a priori analysis - but also demonstrated the development of a cognitive synergy, linked to the alternate use of the two artefacts that promoted the construction of meanings (Faggiano et. al, 2016). The need to examine any changes in order to develop the same path in a "real" class led to the design and implementation of the teaching experiment with the class. The results presented here are based on this last study.

THE INTERACTIVE BOOK AND THE SYNERGIC USE WITH PAPER AND PIN

The artefacts of our duo address the same mathematical content and have been chosen for their semiotic potential, in terms of meanings that can be evoked when carrying out suitable tasks involving their use.

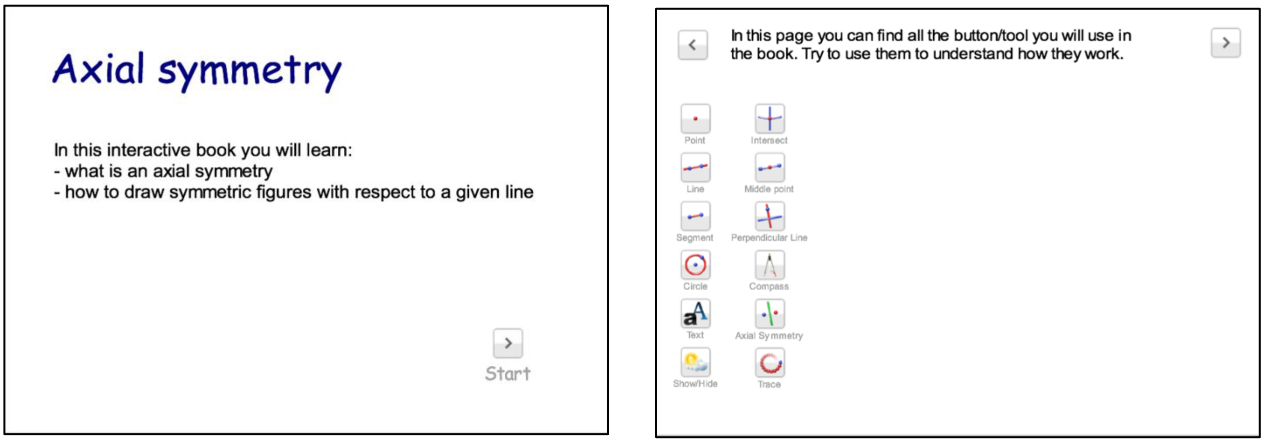
The components of the manipulative artefact are a sheet of paper, with a straight line drawn on it marking where to fold it, and a pin to be used to pierce the paper at a point in order to construct its symmetrical point. This artefact allows an axial symmetry to be created in a direct fashion, because the sheet naturally models the plane and the fold allows the production of two symmetrical points using the pin.

The components of the digital artefact, that appears as an Interactive Book (IB), originate from the components of a specific Dynamic Geometry Authoring Environment (New Cabri - Cabrilog), in which learning activities, involving objects and tools of a DGE, can be created. The IB Book is described with more details in the following section.

The design of the Interactive Book and an overview of the teaching sequence

The Interactive Book has been designed, in order to exploit the potential offered by the dragging function. Below we present, how the design of the tasks embedded in the IB has been developed for the digital artefact and the manipulative one. The main hypothesis inspiring the design concerns the potential synergy between the use of one artefact with respect to the other.

The IB contains a title page (Fig. 1.a) and a page created with the aim to introduce the buttons/tools involved in the activities of the IB (Fig. 1.b). The chosen tools are: those that allow the construction of some geometric objects (Point, Straight Line, Segment, Middle Point, Perpendicular Line, Intersection Point); the "Compass"; the "Symmetry", which gives back a symmetric figure, provided that a figure and a line/axis have been chosen; and the "Trace" tool which, allowing the observation of the relations among the trajectories, makes more evident the effects of the dragging. The next pages of the IB have been integrated in the sequence as it will be explained below.



1.a 1.b

Figure 1. The first two pages of the Interactive Book

In the teaching sequence, in accordance with the study hypothesis, it was decided to alternate activities involving the use of one or the other artefact, formulating tasks that could exploit the complementarity of their semiotic potentials. The sequence, made up of six didactic cycles, begin with the use of the manipulative artefact. It continues with the use of the digital artefact in the second cycle and alternating the use of the artefacts in the third and the fourth cycle, while the order of the artefacts in the last two cycles is inverted.

In the first cycle, pupils are asked to construct the symmetric figure of a given figure with respect to a given line, by folding the paper along the line and piercing with the pin on the necessary points. The acts to fold the paper along the line and to pierce on a point with a pin, in order to obtain a couple of overlapping points, is a first possible way to concretely realise a symmetric configuration.

Such a manipulative experience, can foster the emergence of the idea that an axial symmetry is a one-to-one correspondence between points in the plane, defined by a line, locus of fixed points. In addition, joining the points, obtained with the pin, is the process that yields as product the symmetrical figure, provided that the correspondence between the segments is preserved. This evokes the idea that axial symmetry transforms segments into congruent segments. In the following task of the cycle, pupils are asked to compare what changes and what remains unchanged when drawing two symmetrical figures of the same figure, with respect to two distinct axes. This task has been conceived to evoke the dependence of the symmetrical figure on the axial symmetry.

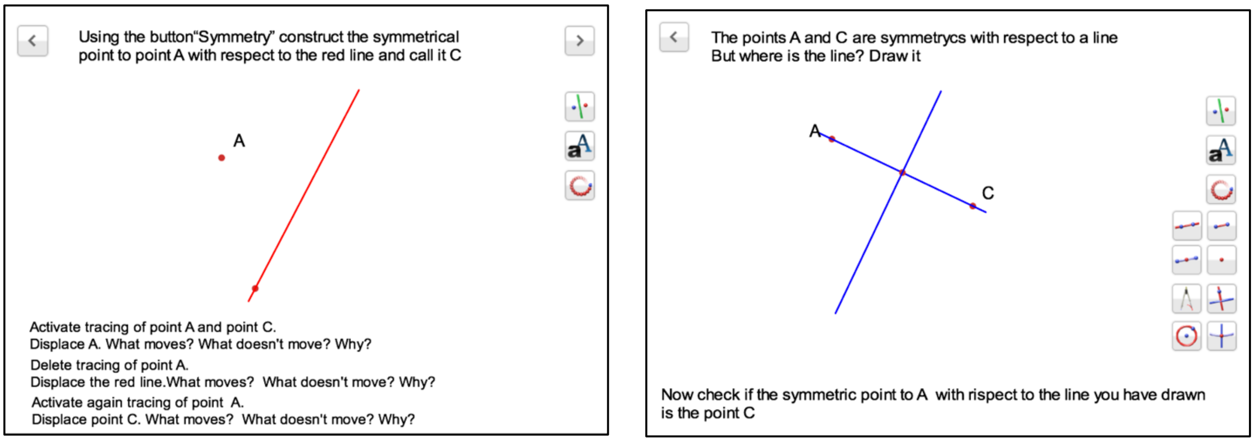
The first activity page of the IB (Fig. 2.a) presents the tasks of the second cycle. They have been designed with the aim to make two key meanings emerge: the dependence of a symmetric point from the point of origin and the role of the line to define an axial symmetry. The pupils are asked to construct the symmetric point of a given point A with respect to a given line, using the “Symmetry” button/tool, and call it C. Then pupils are invited to activate the “Trace” tool on point A and point C, move A and see what moves and what doesn’t, and explain why. In the next two steps, in the same way, the pupils are invited to move the line and the symmetric point and to observe what happens during the dragging.

We emphasize that, in the DGE used, unlike for example in Cabrì Géometrè, it is possible to drag the symmetric point obtained, and this in fact allows the whole paper to be “shifted”.

In this activity, dragging the point of origin and observing the resulting movement of the symmetrical point evokes the idea of the dependence of the symmetrical point on the point of origin; dragging the axis and observing the resulting movement, only of the symmetrical point, evokes the idea of dependence of the symmetrical point on the axial symmetry; dragging the symmetrical point and observing the resulting rigid movement of the entire configuration evokes the idea of the dual dependence of the symmetrical point both on the point of origin and on the axis. The difference in the movements between the symmetrical point and the point of origin can be compared to the distinction between dependent and independent variable.

The tasks of the third cycle aim at: observing that the line joining two symmetrical points is perpendicular to the axis and that the two points are equidistant from the axis; recognizing that these two properties are reversible and that they characterize axial symmetry. With this purpose, pupils are asked to construct the symmetric point without the use of the pin.

The tasks of the fourth cycle are embedded in the next page of the IB. Similarly to the third cycle, pupils are asked to construct the symmetric point without the use of the “Symmetry” button/tool. In order to make this construction, the two properties that characterise axial symmetry, already emerged in the previous cycle, need to be properly used. Pupils, indeed, have to: draw the perpendicular line to the axis, passing through the point of origin; draw the circumference with centre in the intersection point between the axis and the perpendicular line; and finally find the symmetric point as the intersection point between the circumference and the perpendicular line.



2.a 2.b

Figure 2. Examples of activity pages of the Interactive Book

In the fifth and sixth cycles the tasks are the same: there is a pair of points A and C that must be interpreted as symmetrical points with respect to a symmetry where the axis is hidden. Pupils are asked to identify and draw the axis (Fig 2.b). Finally, they are asked to check, using the "Symmetry" button/tool or with the pin, whether the symmetrical point of A with respect to the draw line is really C.

A priori analysis of the potential synergy

The hypothesis formulated is that a reciprocal boosting process will occur, in the form of a synergic process of mediation through the different types of artefacts.

For example, at the second cycle we can expect that the meanings that have already emerged thanks to the use of the manipulative artefact may be extended and completed by the specific meanings that should emerge using the digital artefact.

In other words, the images on the screen can be better interpreted in the light of the previous acts of folding and piercing. In this sense, after having constructed the symmetrical point using the digital artefact, the relation between the two points can be interpreted through the actions of folding, so the two points can be seen as two holes generated by the pin. While the meaning of the relation can be enhanced by the distinction between the original point and the corresponding point, which in the IB corresponds respectively to the direct movement and the indirect movement.

In this way, such a combined interpretation, may contribute to the development of the mathematical meaning of a functional relation between a point (independent) and its symmetrical point (dependent).

Conversely, at the third cycle, we can expect that, the interpretation of the actions and the configurations with the manipulative artefact might be related to the experiences within the digital environment.

For example, we can expect that two different points, of which to construct the symmetric points, can be interpreted as different positions adopted by a point that has been dragged, thereby contributing to the generalization of the two properties (perpendicularity and equidistance) and to the evolution of the status of these properties from being seen as contingent to being seen as characterizing.

Further similar considerations can be done concerning the expected synergy between the artefacts in the next cycles. More details can be found in (Montone et a., 2017).

THE TEACHING EXPERIMENT: RESULTS AND DISCUSSION

In this section we present the analysis of some episodes. In the analysis we attempted to figure out how the use of these two artefacts and their synergy are involved in the construction of the mathematical meanings and the interactions during the discussions.

The first episode refers to the discussion, held in the classroom, at the end of the second cycle after children had used the IB on computers.

During this discussion, one of the children has constructed the symmetric point of a given point with respect to a line, using the IB on the Interactive Whiteboard (IWB). In order to make pupils focus on the diverse movements of the objects on the screen, the teacher asked them:

- to predict what happens if the objects on the screen are moved;
- to verify what happens when they move point A, point C and the line/axis;
- to verify objects' behaviour resulting by the dragging.

In particular, at a certain point the discussion focused on the reason why point C moves and the line doesn't move when dragging point A. The excerpt (Tab.1) concerning M.'s reasoning and V.'s conclusion, is particularly interesting due to the gestures which M. made when speaking.

The importance to refers to the gestures lies in the fact that these signs together with words reveals that the emerging and synergically evolving meanings originate from and remain tied with actions carried out with both the artefacts.

Transcription and *gestures*	**Comments**
M. if you move point A only, point C has to move with point A because they must be symmetrical *M. has her elbows on the desk and moves her hands ahead of her while speaking* 	The objects of M.'s representation move on a virtual space, that is vertical as the screen of the laptop she used, or as the IWB, which is in front of her during the discussion.

like, if you move point A higher… *she raises her left hand to indicate point A moving higher and looks towards her left hand* 	M. accompanies her discourse gesticulating in the space in front of her. These gestures can be considered as pivot signs, because, on the one hand they are related to actions done with the artefact in order to accomplish the task (drag A… in this case “higher”), on the other hand, they are connected, through the feedback of the artefact (point C moves… in this case “lower”), with M.’s sign “the same space”, combined with the gesture.
point C moves lower… so it is the same… *she puts her hands in front of her face, to simulate, with the thumb and index of each hand, two identical segments, she moves her right hand lower to show that, in this case, point C moves lower and looks towards her right hand* 	
…because there must be… the same space… between the two points *with a fast coordinated movement of her hands, she simulates two segments having the same length, using the thumb and index and bending the other fingers* 	The equidistance of the points A and C from the axis, thus, is evoked simultaneously by the verbal sign with the gesture. This sign is again a pivot sign.

V. because there must be the same distance between the line… there must be always the same distance between the two points and the line	The pivot sign "the same space" is evolving into the mathematical sign "same distance".
M. …between the line and the point A and, between the line and the point C	M. recalls what V. said, as to further explain that the distance to be considered is exactly that between each of the points and the axis.
Teacher: Why?	
V. and M. (together) because otherwise they aren't symmetrical!	The equidistance between each of the points and the axis is recognised as a necessary condition for the points to be symmetrical.

Table 1. From "the same space" to "the same distance"

As expected, this episode shows the unfolding of the semiotic potential of the dynamic environment, but also illustrates how the elements used by pupils to support their claims are not limited to refer to the dragging process visualized in the digital artefact. The manipulative artefact appears to be essential to construct the symmetric point and to give rise to a starting conceptualization. However, it gives a static vision because, for instance, after finding a symmetric point of a given point, making a hole on a sheet of paper by piercing it with a pin, the two points cannot move at all. In the previous transcription, instead, M. refers to the dynamic process visualized with the digital artefact: "if you move it", "it moves" and matches words with hand gestures that simulate what she saw on the computer.

The discussion followed-up and the role of synergy emerges: in order to indicate what a symmetric point is, pupils refer to the activity carried out with the sheet of paper and the pin and their initial conceptualization depends on the direct experience of piercing made at the beginning.

Moreover, a further interesting episode which underlines the need to mentally go back to the digital artefact as for G. and the reference in synergy of both artefacts as for V.. The teacher restarts and asks again how they know that the distance is always the same, and G. says:

G.: We figured it out because when [the pupil acting on the IWB] moved point A, point C moved too, but when they were very far away from the red line it was always the distance from the red line… from point C to the red line there was the same distance as… from point A to the red line.

G. matches his speech gesticulating in the space ahead of him. In fact, he looks towards the IWB screen, points his finger towards a hypothetical point A in front of him, with his right hand, while he symmetrically raises his left hand at the same height. He leans back with his body and spreads his arms outwards simulating the two points moving and keeping the same distance from the axis. Here, it shows how the interaction with the digital artefact allowed G. to perceive the invariant element, the distance, thanks to the variation on the screen of the position of point A and consequently of point C, which depends on A. He visually perceives and anticipates the generalization of the invariance of the distance of these two points from the line. In other words, it is as if the pupil visually analysed the variation of an aspect of the whole configuration, keeping another aspect constant, hence anticipating the surfacing of invariant schemes.

Then V., in order to analyse the relationships, explicitly re-calls the manipulative artefact, synergically joins the two activities, and says:

V.: If we have available a sheet of paper that can be folded….

She receives from the teacher a sheet of paper and a pin and makes a symmetric point folding and piercing the paper with the pin, reopens the paper, looks at it, and, simultaneously looking at the IWB adds:

V.: It is more visible there and it is easier… because there you can move the point and so I easily realise that if I move the point… the already created figure… it is easier to realize that there is the same distance because just by moving, you can understand, especially when we distance a lot from the line, that also point C moves… and so there is always the same distance. But I was able to understand it on the paper, also.

V.'s words confirm the hypothesis that the digital artefact is acting in synergy with the manipulative one. However, it is also clear that the modality with which these two artefacts operate is different. The manipulative artefact allows the direct action of the pupil. The pupil's body learns while acting and, in order to describe what a symmetric point is, pupils simulate the folding and piercing of a sheet of paper. When they refer to the digital artefact, instead, pupils simulate with their own body the objects of the actions that they perform with the artefacts: they move the harms as if they were lines and the hands as if they were points, drawing the tracing seen on the screen in the space ahead.

The dragging function, combined with the trace, after allowed pupils to mentally move the objects, and the previous visualization of what happened made explicit the implicit dynamism of thinking mathematical objects.

The next steps show the difference in the way pupils understand that the distance between A and C from the line is always the same: with the manipulative artefact, folding the sheet of paper and observing the superimposition of the two holes; with the digital artefact, animating/moving point A and observing how consequently point C moves. The underlined difference is at the base of the synergic use of the two artefacts since they operate on cognitive processes and different operative and non-superimposable modalities.

CONCLUSIONS

In this paper we have presented the design of a digital artefact, an interactive book developed in a DGE, and of a teaching sequence involving it together with a manipulative artefact. The use of this duo in the teaching sequence was framed by the TSM. The related teaching experiment, conducted with fourth grade students, has been analysed from a semiotic mediation point of view.

The analysis of the results showed, not only the unfolding of the semiotic potential of the artefacts, but also the development of a cognitive synergy, linked to the alternate use of the duo that fostered the construction/conceptualization of axial symmetry and its properties.

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