FEEDBACK IN A COMPUTER-BASED LEARNING ENVIRONMENT ABOUT QUADRATIC FUNCTIONS:

RESEARCH DESIGN AND PILOT STUDY

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Even though feedback is an essential part of computer-based learning environments (CBLEs), it is still not clear what effect different types of feedback have on students math achievement and on their abilities to assess their own achievement. To approach this problem we are currently planning a quasi-experimental study with grade eight or nine students using a CBLE called "Discover Quadratic Functions". This paper gives insight both into existing research concerning the design of CBLE as well as concerning different types of feedback and into the theoretical principals that guided the CBLE design. Additionally, results of a qualitative pilot study as well as the design of the up-coming main study are presented.

Keywords: Computer-Based Learning Environment, Quadratic Functions, Feedback (Response), Mathematics Achievement

THEORETICAL BACKGROUND

The theoretical basis for our research consists of three main topics. Firstly theoretical and empirical findings about CBLEs are reported with a focus on the definition of CBLEs and basic design principles. Secondly, the current state of research regarding feedback is discussed with a focus on the relationship between feedback and achievement and finally the chosen mathematical subject of quadratic functions is explained from a didactical point of view highlighting both mental mathematical representations (*Grundvorstellungen*) and learning difficulties as well as possibilities to avoid or overcome them.

Computer-Based Learning Environments

In search for a definition of CBLEs, it gets obvious that this phrase commonly is used as generic term for computer- and web-based proposals (cf. Baker, D'Mello, Rodrigo, & Graesser 2010; Balacheff & Kaput 1996; Isaacs & Senge 1992). Roth (2015) gives a more precise definition of CBLEs based on mediawiki software. According to him, CBLEs provide structured pathways with a well-matched sequence of tasks, which invites learners to work self-regulated and self-reliantly (Roth 2015). Aside from interactive materials like (GeoGebra-)applets, which are a central content of CBLEs, the integration of retrievable help and the presentation of results are promising ways to support students' working processes in a CBLE (Roth 2015). Wiesner & Wiesner-Steiner (2015) explored central functions of those CBLEs in a qualitative study where they interviewed, among others, experts about central functions regarding the technical and didactical level. Their findings imply that experts gave great account to the (technical) integration of dynamic tools and availability of direct feedback. Concerning the didactic level especially included metacognitive activities and reflexion tasks were pointed out. Suchlike CBLEs are for example available on the German OER-website *ZUM-Wiki* (https://wiki.zum.de/wiki/Hauptseite), which is called on to yield a well-kept surrounding (Vollrath & Roth 2012). The learning environments are subjected to the creative

commons licence CC-by-sa 3.0, so that everybody is invited to copy and even change them on condition that the authors name(s) are mentioned and the licence stays the same. Considerably, all *ZUM-Wiki* CBLE versions are saved online and can be re-activated.

Feedback

There are different models and understandings of feedback. Boud and Molloy (2013) for example distinguish a unilateral from a multilateral view on feedback. In the first, feedback is understood as a "one way transmission" (Boud & Molloy 2013, p. 701) whereby the teacher acts as "driver of feedback" (ibid., p. 698). In contrast, the multilateral view attributes a key role to the learners. This view is shared by several other authors (e.g. Nicol & Macfarlane-Dick 2006, Sadler 1989). For example, Sadler (1989) names feedback only "dangling data" (p. 121), if one does not investigate and monitor if and how feedback effect on students behaviour. In addition, Nicol and Macfarlane-Dick (2006) see a connection between self-regulated work and feedback. Thus feedback "can help students take control of their own learning, i.e. become self-regulated learners" (Nicol & Macfarlane-Dick, p. 199). According to this, they worked out seven principles of good feedback practice, such as "[it] facilitates the development of self-assessment (reflection) in learning" (Nicol & Macfarlane-Dick, p. 205). Beside the feedback *practice*, the possible *types* of feedback are of interest for our research. The complexity of feedback is called a specific aspect of effective feedback, whereby studies about this particular field have yielded inconsistent results (Mory 2004, cf. Nelson & Schunn 2008, Shute 2008). The impact of feedback for the process of learning is investigated frequently (cf. Black & Wiliam 1998; Shute 2008; Hattie 2009). Hattie (2009) for example identified feedback as one of the top influences on achievement in school. The intensity of this influence differs depending on the kind of provided feedback. While *knowledge of the correct response feedback* (KCR) has shown similar effects to *no feedback*, variants of feedback that have a *multiple-try* function (MTF) have shown a positive gain on learning (Attali 2015; cf. Shute 2008; Niegemann 2008). MTF offers the possibility to re-think results and thus to remove mistakes. Provided additional help may structure and foster this processes (Attali 2015). Furthermore, Attali (2015) names the effects of providing "explanations for the correct answers" (p. 266), which means a combination of KCR and explanations, as "an interesting area for future research" (p. 266). Dempsey, Driscoll and Swidell (1993) have defined this kind of feedback earlier and allocated it as *elaborated feedback* (EF). Kulhavy and Stock (1989) categorised basically three elaboration types: "(a) task specific, (b) instruction based and (c) extra-instructional" (p. 286). Shute (2008) offers some more types of EF such as giving hints that guide the learners. She says that in general EF "provides information about particular responses or behaviours beyond their accuracy" (Shute 2008, p. 157). In addition to the portrayed variants of feedback, reference should also be made to other influencing factors such as prior knowledge and the point of time when feedback is given (Shute 2008, cf. Mory 2004). The research outcomes concerning the proper time for feedback are divergent, although different meta-analysis tend to foster immediate feedback (cf. Bangert-Drowns et al. 1991; Shute 2008; Niegemann 2008).

Quadratic Functions

Quadratic functions play a central role in German secondary math education (NRW Ministry for Schools and Further Education 2007) and there is wide range of didactical considerations of this topic. One important current revolves around the idea of mental mathematical representations, in German *Grundvorstellungen*. Doorman, Drijvers, Gravemeijer, Boon, and Reed (2012) mention three of them concerning the function concept: "functions as an input-output assignment", "functions as a dynamic process of co-variation" and "functions as a mathematical

object" (p. 1246). Besides, many others have specified these concepts as well (e.g. Vollrath 1989; Malle 2000; Greefrath, Oldenburg, Siller, Ulm, Weigand 2016). The input-output concept refers to the attribution of a domain element to a single element of the target set, co-variation records how changes of one quantity involve modifications of another one and object perceptions means seeing functions as a single object, which describes a relationship as a whole. Functions as mathematical objects are primarily discussed in German upper school (Greefrath et al. 2016). Hence, lower secondary education especially addresses the other two concepts. Zaslavsky (1997) indicates five "cognitive obstacles" (p. 20) concerning quadratic functions (Zaslavsky 1997, p. 30–33):

> "<u>Obstacle 1.</u> The interpretation of graphical information (pictorial entailments) […]
> <u>Obstacle 2.</u> The relation between a quadratic function and a quadratic equation […]
> <u>Obstacle 3.</u> The analogy between a quadratic function and a linear function" […]
> <u>Obstacle 4.</u> The seeming change in form of a quadratic function whose parameter is zero […]
> <u>Obstacle 5.</u> The over-emphasis on only one coordinate of special points".

Furthermore, Nitsch (2015) reports about learning difficulties in the field of representational changes upon functional relationships. One of the difficulties she reveals within her study refers to the understanding of parameter impacts on the graphical representation of quadratic functions (Nitsch 2015). Systematic variation of parameters may be a connecting factor to foster understanding (cf. Vollrath & Roth 2012). It can for instance be offered by the possibility to use a slider in dynamic geometry applets.

RESEARCH QUESTIONS

Based on the portrayed research findings regarding variants of feedback and in view of the possible learning difficulties in dealing with quadratic functions, it is interesting to investigate the impact on self-rating as well as achievement.

1. Is students' self-rating better if they work with a CBLE including feedback towards the correct solution in combination with explanations, than it is when students receive feedback without explanations (research based on quadratic functions)?

2. Does a computer-based learning environment, including feedback towards the correct solution in combination with explanations, have greater benefit on students' math achievement in comparison to students' achievement when they receive feedback without explanations (research based on quadratic functions)?

In preparation for the main study, in which the research questions above will be examined, a pilot study pays attention to the evaluation and enhancement of the designed CBLE about quadratic functions.

0.1 How do students perceive their self-reliant work with the CBLE? What do they think about the given steering measurements?

0.2 Which metacognitive contents of the CBLE are estimated supportive by students for the process of learning?

DESIGN

In the following sections, two different types of design are described. On the one hand the research design and on the other hand, the underlying thoughts regarding the designed CBLE about quadratic functions are illustrated. The research design, especially the main study design, depicts our plans
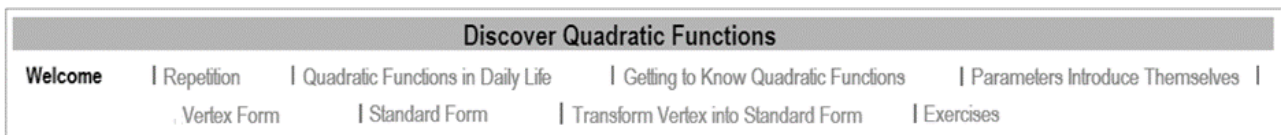
and might be adapted due to supplementary requirements. Currently we are conducting some preliminary studies with various focuses. Concerning the CBLE design, some general reflections, exemplary variants of integrated feedback and two inserted exercise formats are described in the following paragraph. In our research the definition by Roth (2015), concerning CBLEs based on mediawiki software is used.

Research Design
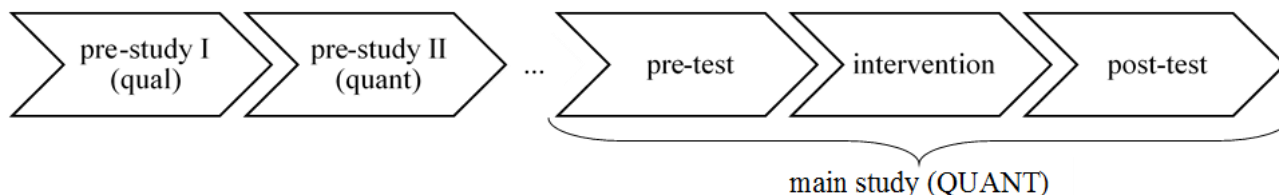
The research questions already implicate that the main study will focus on different types of feedback, which can be integrated in CBLEs, and especially on their influence on students' math achievement. As a further interest, we want to examine if there is a link between received feedback and self-assessment in the sense of self-rating the own achievement. A quasi-experimental study with a control group design (1x1) is planned. Both the control and the experimental group receive the same CBLE and self-assessment scales. Since the scales have to be in accordance with the CBLE contents, no pre-existing questionnaire can be used and the scales are going to be developed too. The experimental group receives immediate feedback about the correct answer of a task included in the CBLE. This feedback combines KCR with explanations and a kind of MTF. It has to be discussed if the included feedback can be ranked as EF. The control group gets another variant of feedback in CBLEs, comparable to KCR, which mainly means that there are no explanations about the procedure of solution and no prompts to try a task again. Our aim is to examine if the type of feedback in the experimental group is as effective as the theory leads us to think. As mentioned above, KCR has shown similar effects to giving no feedback, whereby the combination with explanations and a multiple-try function may show positive effects on learning (p. 2). But since the risk remains that students use the feedback in a nonreflective way and just copy the prompted results, it is also possible that the experimental group's gain in math achievement stays behind the control group's. To measure the math achievement before and after students' work in the CBLE, we are developing two tests that are connected via anchor items. While in the pre-test the focus lays on functional thinking and linear functions with only a few items on quadratic functions integrated to measure previous knowledge, the post-test contains items measuring functional thinking as well as knowledge about quadratic functions. Those items are designed to be comparable to the pre-test items concerning linear functions. A duration of six school lessons for the intervention is intended between pre- and post-test. Learners are supposed to work on their own respectively in groups of two students with the CBLE. Teachers' role is to be attendant as advisor especially for technical questions, but stay out of students' work in total. Before the conduction of our main study, we are conducting several preliminary studies. On the one hand, we conduct qualitative pre-studies to test and enhance the designed CBLE (pre-study I). One of these studies with focus on students' working experience in the CBLE is described below. Others will be expert interviews as well as monitoring students' while working with the entire CBLE. On the other hand, a quantitative pilot run is planned to check the quality of the used achievement test as well as to get a first impression of what kind of learning progress can be expected in the main study (pre-study II).

Computer-Based Learning Environment *Discover Quadratic Functions*

| Discover Quadratic Functions				
Welcome	Repetition	Quadratic Functions in Daily Life	Getting to Know Quadratic Functions	Parameters Introduce Themselves
	Vertex Form	Standard Form	Transform Vertex into Standard Form	Exercises

Figure 1. Index of the created CBLE *Discover Quadratic Functions***. (Translated)**

At the moment, the CBLE consists of eight chapters shown in figure 1. After a short introduction (*Welcome*), which combines technical instructions with a declaration of required- and goal-competencies, the learners are encouraged to work with the CBLE. If they are not sure about whether they already possess the needed competencies, they have the opportunity to work on tasks that repeat basic knowledge (*Repetition*). Otherwise, they may enter the next chapters *Quadratic Functions in Daily Life* and *Getting to Know Quadratic Functions*. The formers thematic priority is motivation, whereby the latter introduces simple quadratic functions ($f(x) = x^2$). The next step is to work with parameters (*Parameters Introduce Themselves*) and thus to discover the *Vertex Form.* Afterwards the *Standard Form* is thematised as well as the proceeding to *Transform Vertex into Standard Form.* The CBLE ends with further *Exercises* about all included subjects. Within one chapter, the learners can decide about the order of the exercises and the time they spend on each. Regarding the interactive tasks, a repetition is possible too (cf. figure 2). Furthermore, within some application tasks the learners can adapt the level of difficulty (cf. figure 3). Nevertheless, since this CBLE introduces a new subject area for learners, the flexibility is limited in contrast to CBLEs that are built for e.g. repetition. Besides to working at the computer, students are needed to write into a notebook while working with the CBLE. Some tasks explicitly ask for paper-pencil work ensuring that learners do not forget how to draw a graph by hand for example. In addition, students may gather mnemonic sentences or complete self-assessment scales in there.

There are different exercise formats integrated in the CBLE. Besides being interactive or not, the exercises can be distinguished into inner-mathematical or application tasks. The inner-
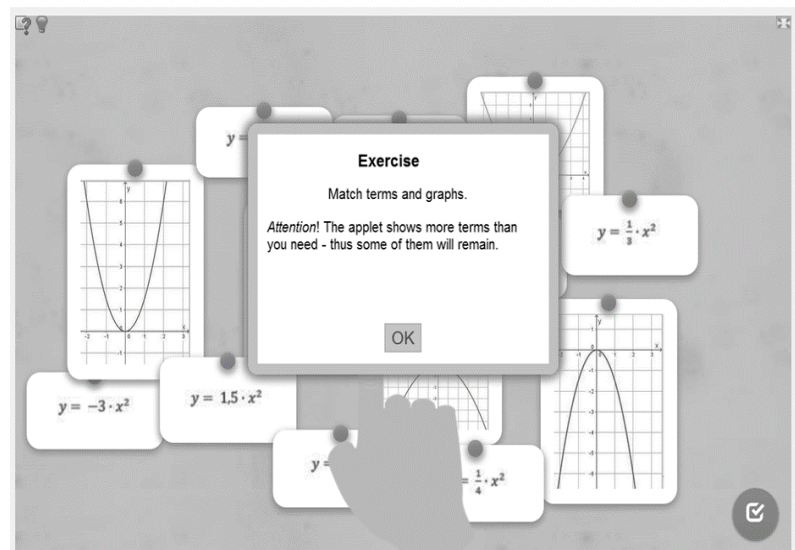
Figure 2. Exemplary inner-mathematical task about matching terms and graphs. (Translated)

a) Think about ball sports (or something comparable) whose ball trajectories can approximately be explained ba a parabola. Write down a suitable term - without talking to your partner. The GeoGebra-Applet beneath may help you to visualize it.

Show Hints

b) Try to detect the ball sports he or she meant to describe. The GeoGebra-Applet as well as the Hints may help you. When both of you have a solution, talk to each other and reflect you conclusions.

Figure 3. Exemplary application task about quadratic functions. During the exercise, students need to work in pairs. (Translated)

mathematical tasks serve as an introduction to get to know the subtopics of quadratic functions and to consolidate new skills. The application exercises are included to deepen students' knowledge. In figure 2, an inner-mathematical task of the CBLE is shown. It exposes an interactive applet in which the learners should match quadratic terms and parabolas. After they have finished, they can check their results by clicking the button downright. Correct answers are marked green, wrong ones red. If no matched pairs remain, their solution is omitted. This exercise is also exemplary for allowing and encouraging multiple tries. Application tasks need more skills than draw on taught issues. Figure 3 for example requires knowledge about how to modify quadratic functions and creativity as well. The shown exercise is about finding a suitable term for a freely chosen ball sports. In the following tasks the learners exchange terms with a partner who tries to detect the underlying sports. Last step is to explain ones decisions and to reflect it together in pairs.

Since the CBLE is built to investigate the impact of feedback during the main study, figure 4 shows different variants of feedback which are applied. The rationale behind choosing exactly these feedback variants are mainly due to the possibilities delivered by *ZUM-Wiki*. On the one hand, interactive applets are bound. As mentioned above, this kind of tasks has a control button whose activation shows if the entered solution is right or should be reconsidered (figure 4a). On the other hand, hints and solutions are integrated. They are hidden until the learners activate them by mouse click (figure 4b). For further information about the concrete kinds of feedback behind it and the rationale for using these variants, see paragraph "Research Design" (p. 4).
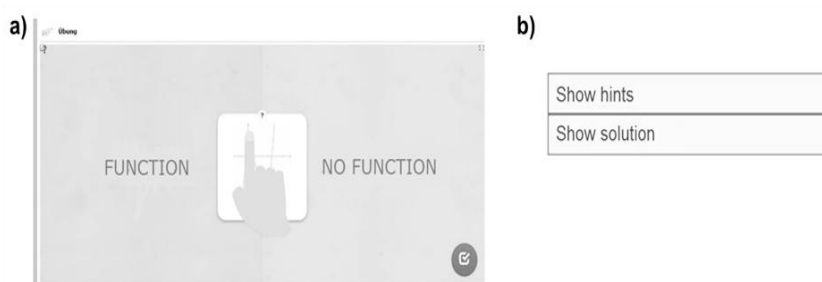
Figure 4. Exemplary variants of feedback that are included in the designed CBLE: a) Bound interactive applet with a control button. b) Hidden help and solution. (Translated)

PILOT STUDY

In preparation for the main study, the designed CBLE is tested. The portrayed pre-study is qualitative with the aim to evaluate and enhance the designed learning environment (pre-study I). It has been performed in cooperation with Sur (2017) in the course of his master thesis at the University of Münster. Six ninth grade students of a high school (Gymnasium) in North Rhine-Westphalia have participated within this study. They worked for approximately 45 minutes with the CBLE chapter *Vertex Form* [1] and were afterwards questioned in guided interviews with a duration of about 15 minutes each. The interviews were transcribed and coded with MAXQDA. The coding was based on summarizing content analysis according to Mayring (2010), whereby the used procedure can be declined to have a focus on inductive coding with some deductive approaches.

The guided interviews contained questions related to research questions no. 0.1 and 0.2 (p. 3). The questions were verbalised in an open way that animated the students to speak freely. For example, they were asked: "You have now worked on your own with the CBLE for 45 minutes. Which parts

of the CBLE supported your work? a) How did you use it? b) In which way did it support your working process?" (Sur 2017, p. XV, translated).

FINDINGS AND DISCUSSION

Students' comments were summarized to the intended categories *steering measurements and participant activity* (combined to the generic term *self-reliance*) and *metacognition*. These two categories concern research questions no. 0.1 and 0.2 and findings relying on them are herein depicted in brief. Since the results of the pilot study are organised in categories, their depiction will follow the same structure. First some remarkable citations are shown (translated; names modified). Afterwards positive aspects the students' named as well as some of their supplementary remarks are summarized. With reference to the rationale for research questions 0.1 and 0.2, each paragraph has integrated some exemplary résumé on how the CBLE will be revised due to the presented results. As final remark it should be emphasized that the shown findings just express the individual meaning of a small group of students (n=6). There is no aspiration to generalize them, but they serve as a starting point for a complete evaluation of the developed CBLE. Besides they are going to be supplemented by expert interviews.

Self-Reliance

Isabell: I liked to work self-reliantly and yes it is something different from only being present in classroom and absorb thinks like a sponge.

Felix: This partner work. That was good; it was not working all by myself, but to have the possibility to compare how others work.

According to their own statements, students liked to work actively with the contents of the CBLE. They also welcomed the variability of some tasks, for example, when it was their turn to choose three of five pictures to work with (according CBLE chapter [1], exercise 1). Occasionally, students wished to be more assisted by the teacher, especially at the beginning. Tasks, which included the need to work in pairs accommodated the students and have been highlighted (cf. Felix' citation). In addition, the immediate feedback was mentioned as being helpful. One of the students remarked that she felt pressed for time.

With reference to the last point, we are going to provide a weekly schedule in future. It is to be hoped that this overview will facilitate time-management. Assistance by teacher is difficult to manage during the quantitative study, because of non-evaluable influences. A more detailed briefing in combination with extended CBLE-usage, as well as the according weekly schedule may perhaps promote the students' self-reliant working processes. Since the students are going to work self-reliantly for a long time during the main study, it may be thought of organizing them in pairs for the whole time.

Metacognition

Mia: I liked the possibility to self-control my results.

Marcus: I see slight risks because of the integrated feedback. Perhaps one looks immediately for the solution.

Metacognitive components of the CBLE are the integrated transparent goals, self-assessment scales, hints and suggested solutions. Students mainly underlined the transparent goals. They named them helpful for understanding why to work on the following tasks. The self-assessment scales have been presented as open questions, which is why some of the students have had some phrasing difficulties

as they stated. Nonetheless, this activity was highlighted assistant for the process of learning and for teachers' acknowledgement as well. Students in this study made not much use of hints. A probable reason is that the learners already knew quadratic functions and used the CBLE chapter for repetition. However, they remarked that the hints might be an enrichment for students who are getting to know quadratic functions. Finally, students had different opinions on the suggested solutions. Most students liked the possibility to self-control their results (cf. Mia's citation), but saw associated risks as well (cf. Marcus' citation). According to the students' statements, detecting mistakes was facilitated because of the retrievable feedback.

Regarding these findings, we are going to transfer the self-assessment scales into a closer format, e.g. a checklist. On the one hand, we thereby hope to foster the students' self-rating activity. On the other hand, it will also be easier to analyse the filled scales by statistical means. The mentioned risk, namely using the feedback unreflectively, is exactly what we want to investigate in our main study.

SUMMARY AND PROSPECT

Within this paper, we reported on preparations we made for an upcoming study about the impact of different kinds of feedback integrated in a CBLE about quadratic functions on students' math achievement and self-assessment. Framed by the theoretical background, we drew up our research questions and informed about the planned main study as well as about several pre-studies. Furthermore, design choices concerning the content of the CBLE and the included feedback variants have been presented. Thus far, we conducted a pilot study with the aim to evaluate and enhance the CBLE (pre-study I). Based on the findings of the reported pilot study and to gain more information about how our adaptions and assumptions work in total, further qualitative pre-studies will accompany the quantitative pilot-run (pre-study II) which will take place in November 2017.

NOTES

1. See ZUM-Wiki link: https://wiki.zum.de/wiki/Quadratische_Funktionen_erkunden/Die_Scheitelpunktform (in the version of 2016-11-29), only available in German language.

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