E-assessment is a powerful tool for supporting the learning of mathematics. Early trials began back in 1991 on a local network. Over the past 25 years technical advances have widened and improved its delivery. For the past 10 years MapleTA has been adopted and a huge range of question banks have been developed. Some recent topics include dimensional analysis, fractals, linear programming, Fourier series, oscillatory motion, series solution of ODEs, Laplace transforms and basic solution methods for PDEs. A key element in providing students with feedback on their progress is the “How Did I Do?” option, which allows them to check their answers as they progress through extended problems. The same question is equally relevant when evaluating the effectiveness of e-assessment for many thousands of students over several decades.

Keywords: E-assessment, feedback, modelling, mechanics, calculus

A QUARTER CENTURY OF E-ASSESSMENT ADVANCES

It is over 25 years since e-assessment was first used for mathematics students at the University of Portsmouth on a local network (Figure 1).

![Figure 1. Mathematics e-Assessment Delivery using QuestionMark Software 1991-2007](image)

Although computer algebra systems were not widely available then, it was still possible to author and deliver a variety of standard question types. In 1996 a CAS powered assessment system was developed (McCabe and Watson, 1997) using the Maple kernel within Toolbook authoring software (Figure 2). For the first time ever it was possible to check algebraic question responses with a CAS and develop mathematical questions with a user-friendly interface.

Around the same time the delivery of online assessment was beginning and the main tool used at Portsmouth was QuestionMark Perception (McCabe, 1998), which had no underlying CAS. In 2005 the Department of Mathematics switched to using MapleTA and it has been the primary tool used for e-assessment since then (Figure 3). Initially a local server was used, but in recent years a managed server has proved more convenient and reliable, especially when dealing with product upgrades. Increasing student numbers at Portsmouth (McCabe 2009) made the effort worthwhile.
The adoption of a commercial product, rather than an open source e-assessment system, such as STACK (Sangwin, 2004), Numbas (Foster, Perfect and Youd, 2012), DEWIS (Gwynllyw and Henderson, 2009) and Math e.g.(Greenhow and Kamavi, 2012), has provided stability and the availability of support when it was required.

AN EVOLVING STRATEGY FOR E-ASSESSMENT DELIVERY

The literature on e-assessment has grown considerably over the past 25 years. Timmis et al (2016) provides an up-to-date set of general references for what it calls Technology Enhanced Assessment TEA. Sangwin (2013) is the first textbook specifically on the subject of mathematics e-assessment and many other sources of guidance on e-assessment have been written over the years, e.g. Whitelock (2006), QCA (2007). At Portsmouth it has largely been years of practice and a gradual evolution that has shaped the present strategy for delivering e-assessment.

E-assessment delivery initially focussed on summative tests. Mathematical Models is a typical 1st year mathematics undergraduate course unit, for which MapleTA has been routinely used over the past 10 years. As question banks have increased in size, weekly practice tests with feedback have become the norm. A monthly coursework assessment on each topic, allows students a controlled 24-hour period to complete their work. Although different assessment patterns have been tried out, our experience is that a 40:60 weighting of continuous assessment to a final exam motivates students to work steadily through a course unit and achieve high marks as they progress. The final e-assessment exam lasts 2-hours and is always formally invigilated. Intermediate Calculus, a 2nd year course unit, adopts a similar progressive style of weekly practice e-assessments, monthly 24-hour coursework, but with a more traditional 2-hour written final exam. The weekly practice assessments often promote flipped learning, with many students using them as the starting point in their study.
EFFICIENT DEVELOPMENT OF NEW QUESTION BANKS

The efficient production of high quality algorithmic questions with feedback has been the key to the successful delivery of e-assessment. The many features of MapleTA have enabled rapid authoring without getting bogged down in technicalities. Three special cases are highlighted here: reverse engineering, randomised components (datasets, functions, equations, graphs, networks, matrices ...) and multipart questions.

Figure 4. Efficient Question Setting Via Reverse Engineering of Dimensional Analysis

Special Case 1: Dimensional analysis is an extremely useful mathematical technique for solving problems with minimal work, but without a full understanding of the underlying physical processes. It is introduced as part of the Mathematical Models course unit. The left hand screenshot in Figure 4 shows a typical “real-world” question, which leads a student through the solution of a specific problem. The drawback is that finding sufficient realistic dimensional analysis problems to solve and the creation of a question bank is time-consuming. To avoid this, a randomised set of fictitious problems have been developed which allow the technique to be practiced effectively on meaningful questions. To illustrate its implementation, suppose we wish to find an unknown relationship

\[ X = X(A,B,C) = kA^nB^mC^p \]

If the dimensions of \( X, A, B \) and \( C \) are given as \( M^aL^bT^c, M^aL^bT^c, M^aL^bT^c, M^aL^bT^c \) respectively, then we deduce that

\[
\begin{align*}
    a_1 n + b_1 m + c_1 p &= x_1 \\
    a_2 n + b_2 m + c_2 p &= x_2 \\
    a_3 n + b_3 m + c_3 p &= x_3
\end{align*}
\]

We could solve for \( n, m \) and \( p \) using Maple, but cannot easily be assured of user-friendly solutions. Instead the trick is to reverse engineer the question. Rather than setting up a randomised question and solving it, we start with a randomised solution for \( n, m \) and \( p \), but then create randomised questions by choosing suitable question parameters. In practice, this simply means randomising \( a_i, b_i \) and \( c_i \) (\( i=1..3 \)) and calculating \( x_1, x_2 \) and \( x_3 \) from the three equations shown above. An
important condition, easily implemented in a MapleTA question algorithm, is to ensure that
\[
\begin{bmatrix}
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2 \\
a_3 & b_3 & c_3
\end{bmatrix} = 0
\]
is satisfied. A resulting question is shown on the right hand side of Figure 4. The difficulty of the
question is readily adjusted by varying the range of randomised parameters and adding more
physical quantities, such as temperature. Reverse engineering is an important technique in setting
e-assessment questions in mathematics and allows them to be generated far more simply and
reliably than the direct approach of solving a randomly generated problem.

Special Case 2: Algorithms with randomised components lie at the heart of most questions. The
generation of simple numerical datasets for fractal box counting is a classic example (Figure 5). A
simple logarithmic relationship \( \log N(s) = \log C + D \log s \) implies that a graph of \( \log N(s) \) vs. \( \log s \)
will be a straight line with slope \( D \), the fractal dimension. Data can be generated with or without
randomised “noise”.

Figure 5. Randomised Datasets for Fractal Box Counting

Large banks of ODE and PDE questions have been developed with randomly generated
equations, covering a wide range of types and solution methods.

Figure 6. Randomised Equations for ODE and PDE Solution
For these questions, the technique of cloning, i.e. the copying and modification of existing questions, plays an important role in speeding up question production. Often only minor changes are needed to generate a completely different question.

Randomised graphs can be generated in MapleTA very efficiently. The matching question in Figure 7 is an example taken from a linear programming question bank and includes a different, graph for each of the 4 solution possibilities. Any graph that can be generated in Maple can be randomised in MapleTA with minimal effort.

![Figure 7. Randomised Graphs for Linear Programming](image)

Other examples of efficient graph plotting, using Maple commands and packages, are shown in Figure 8 below. All the graphs are generated dynamically for each instance of the question with a single command.

![Figure 8. Randomised Graphs for Networks and Parametric Coordinate Problems](image)
Special Case 3: Multi-part questions are used frequently to guide students through common solutions methods. Figure 8 (right) shows the combination of randomised graphs with a multipart question in solving a problem involving parametric coordinates. A further example, shown in Figure 9, is a question which works through statistical hypothesis testing.

Figure 9. Multi-Part Question for Statistical Hypothesis Testing

Without efficient means of authoring e-assessment questions, their development becomes a slow and unproductive process. The process can become even slower when the time taken to add feedback is taken into account. Experience has shown that reverse engineering, randomised algorithmically generated components and multi-part questions are often the key elements in creating effective, reusable questions.

EFFICIENT AND EFFECTIVE DELIVERY OF FEEDBACK

Basic principles of feedback practice in e-assessment have been well identified over the years, e.g. Nicol and Milligan (2006), but are often extraordinarily difficult to implement. The preparation of detailed, dynamically generated feedback can be extraordinarily time-consuming and may often take longer to produce than the question itself. It is often found to be more efficient to provide traditional written solutions to sample questions. Indeed it can be argued that automated solutions and feedback can eliminate the need to cross-reference different sources of information, such as textbooks, lecture notes and worked examples, making the arena of learning too restricted.

Although students are encouraged to read their lecture notes and worked examples, when they tackle e-assessment questions, some automatically generated feedback is always worthwhile. In MapleTA students can simply ask “How Did I Do?” while they are attempting a multi-part question. For example, when solving an ODE using Laplace transforms (Figure 10), partial solutions can be checked before moving on to later stages of the solution. Thus, mistakes in early parts of the question can be corrected before moving on to the complete solution.
Often a compromise must be reached between time spent on preparing fully automated feedback and more questions for a bank. The pragmatic approach has been to use the “How Did I Do?” option to provide a basic level of feedback without requiring much extra work. When it is routinely available in all questions during weekly practice tests, students usually take full advantage of the help that it gives them.

![Laplace Transform Problem](image)

**Figure 10. How Did I Do on my Laplace Transform Problem?**

**THE PRESENT AND FUTURE OF E-ASSESSMENT**

Mathematics e-assessment has advanced considerably over a period of more than 25 years, since it was first used at the University of Portsmouth. Software developments have enabled online delivery, CAS checking of responses, randomisation in many different forms, algorithmic question generation, multi-part questions, new question types, targeted feedback and adaptive questions. Changes in the software tools over the first 15 years often made it necessary to abandon existing question banks and write new ones. Maintenance and improvement of question banks is still important, but far less time-consuming than it used to be. For the past 10 years there has been relative stability and the size of question banks has grown (Figure 10) as new topics have been added. With the recent addition of Fourier series, the total number of MapleTA questions now approaches 1000. These are available for MapleTA users of the future, who can also answer the question “How Did I Do?”

![Megabank of MapleTA Questions](image)

**Figure 10. A Megabank of MapleTA Questions**
REFERENCES


