25 YEARS OF E-ASSESSMENT AND BEYOND: HOW DID I DO!

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E-assessment is a powerful tool for supporting the learning of mathematics. Early trials began back in 1991 on a local network. Over the past 25 years technical advances have widened and improved its delivery. For the past 10 years MapleTA has been adopted and a huge range of question banks have been developed. Some recent topics include dimensional analysis, fractals, linear programming, Fourier series, oscillatory motion, series solution of ODEs, Laplace transforms and basic solution methods for PDEs. A key element in providing students with feedback on their progress is the "How Did I Do?" option, which allows them to check their answers as they progress through extended problems. The same question is equally relevant when evaluating the effectiveness of e-assessment for many thousands of students over several decades.

Keywords: E-assessment, feedback, modelling, mechanics, calculus

A QUARTER CENTURY OF E-ASSESSMENT ADVANCES

It is over 25 years since e-assessment was first used for mathematics students at the University of Portsmouth on a local network (Figure 1).

Date(s)	Development/Activity/Action	Outcome(s)	Software	Focus
1991	Installation of networked Computer Assisted Assessment (CAA)	First use of summative e-assessment	Question Mark for DOS	Departments
1994	Upgrade of networked CAA to Windows	Delivery of improved functionality, especially graphics/ graphical questions	QM Designer for Windows	Departments
Mid-1996	Beta testing of on-line e-assessment	Feasibility of on-line e-assessment demonstrated	QM Perception Beta + PWS	Dept of Mathematics
Late 1996	Installation of on-line e-assessment Full-time educational technologist appointed	Successful first use of on-line summative e-assessment	QM Perception V1 + PWS	Dept of Mathematics
1997 - 1998	University Project: CAA on the Web (£4500) Purchase of 10 QMP personal licences	Successful pilot use of on-line e-assessment by 10 individual staff covering all faculties	QM Perception V1 + Personal Web Server	University
1999 - 2000	University Project: Framework for the Uptake of CAA (£10,500)	Successful scaling up of on-line assessment with 2 faculty servers Use of WebCT connector proposed	QM Perception V2	University Faculties
April 2001	Proposal for university wide Perception site licence presented to university IT committee	Proposal accepted	QM Perception V3	University
Nov 2001	QM Perception purchased (£25,000) Ongoing annual maintenance (£10,000 per annum)	Full site university licence available	QM Perception V3	University
2002	Full implementation strategy document	Unsuccessful trial set up in Faculty of Technology	QM Perception V3	Faculty of Technology
2003	Purchase of NT server for e-assessment (£5000)	Re-installation of QM Perception	QM Perception V3.4	Faculty of Technology
Mid-2004	Use of Oracle database for storage of questions, assessments and results	Successful trial of full scale Oracle based e-assessment system	QM Perception V3.4 + Oracle	Faculty of Technology
Sept 2004	University On-Line Learning and Assessment Group sets up full-scale pilot	Pilot delivery of diagnostic & summative assessments in mathematics	QM Perception V3.4 + Oracle	Department
2005	Report of successful pilot Oracle based Perception 3.4 Academic PVC identifies Perception as university tool	Use of Web based Perception extended to Science Faculty	QM Perception V3.4 + Oracle	University
2005	Installation of QM Perception V4		QMPV4 + Oracle	University
Oct 2005	Proposal to include e-assessment in academic regulations	Regulations modified		University
2006	Conversion of assessments to QMP V4	Increasing number of assessments delivered	QMPV4 + Oracle	Technology
2007	Use of QM Perception by four Faculties: Technology, Science, CCI and HSS	Successful delivery of formative and summative assessments	QM Perception V3, V4 + Oracle	University

Figure 1. Mathematics e-Assessment Delivery using QuestionMark Software 1991-2007

Although computer algebra systems were not widely available then, it was still possible to author and deliver a variety of standard question types. In 1996 a CAS powered assessment system was developed (McCabe and Watson, 1997) using the Maple kernel within Toolbook authoring software (Figure 2). For the first time ever it was possible to check algebraic question responses with a CAS and develop mathematical questions with a user-friendly interface.

Around the same time the delivery of online assessment was beginning and the main tool used at Portsmouth was QuestionMark Perception (McCabe, 1998), which had no underlying CAS. In 2005 the Department of Mathematics switched to using MapleTA and it has been the primary tool used for e-assessment since then (Figure 3). Initially a local server was used, but in recent years a managed server has proved more convenient and reliable, especially when dealing with product upgrades. Increasing student numbers at Portsmouth (McCabe 2009) made the effort worthwhuile.

The adoption of a commercial product, rather than an open source e-assessment system, such as *STACK* (Sangwin, 2004), *Numbas* (Foster, Perfect and Youd, 2012), *DEWIS* (Gwynllyw and Henderson, 2009) and *Math e.g.*(Greenhow and Kamavi, 2012), has provided stability and the availability of support when it was required.

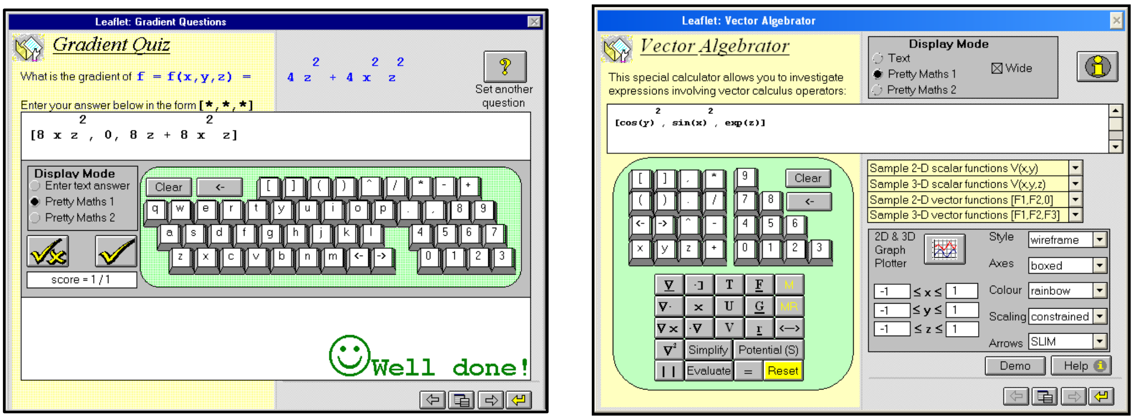


Figure 2. World First Use of CAS for e-Assessment (McCabe and Watson, 1997)

Date(s)	Development/Activity/Action	Focus	Status
Early 2005	MapleTA identified as an appropriate e-assessment tool for maths, science and technical subjects	Department	Investigation
May 2005	MapleTA V2.5 ordered	Department	Investigation
July 2005	Software installed	IS	Testing
Sept 2005	Minor problems rectified	IS	Testing
Oct 2005	Authoring begins	Department	Testing
Nov 2005	First successful delivery of a summative assessment	Department	Pilot
March 2006	Successful delivery of formative and summative assessment on two maths units	Department	Production
June 2007	Successful delivery of formative and summative assessment on four units: 2 maths, 1 science and 1 civil engineering	Faculty	Production
October 2007	MapleTA upgrade to V3	IS	Investigation

Figure 3. Early Mathematics e-Assessment Delivery using MapleTA 2005 - 2007

AN EVOLVING STRATEGY FOR E-ASSESSMENT DELIVERY

The literature on e-assessment has grown considerably over the past 25 years. Timmis et al (2016) provides an up-to-date set of general references for what it calls Technology Enhanced Assessment TEA. Sangwin (2013) is the first textbook specifically on the subject of mathematics e-assessment and many other sources of guidance on e-assessment have been written over the years, e.g. Whitelock (2006), QCA (2007). At Portsmouth it has largely been years of practice and a gradual evolution that has shaped the present strategy for delivering e-assessment.

E-assessment delivery initially focussed on summative tests. Mathematical Models is a typical 1st year mathematics undergraduate course unit, for which MapleTA has been routinely used over the past 10 years. As question banks have increased in size, weekly practice tests with feedback have become the norm. A monthly coursework assessment on each topic, allows students a controlled 24-hour period to complete their work. Although different assessment patterns have been tried out, our experience is that a 40:60 weighting of continuous assessment to a final exam motivates students to work steadily through a course unit and achieve high marks as they progress. The final e-assessment exam lasts 2-hours and is always formally invigilated. Intermediate Calculus, a 2nd year course unit, adopts a similar progressive style of weekly practice e-assessments, monthly 24-hour courseworks, but with a more traditional 2-hour written final exam. The weekly practice assessments often promote flipped learning, with many students using them as the starting point in their study.

EFFICIENT DEVELOPMENT OF NEW QUESTION BANKS

The efficient production of high quality algorithmic questions with feedback has been the key to the successful delivery of e-assessment. The many features of MapleTA have enabled rapid authoring without getting bogged down in technicalities. Three special cases are highlighted here: reverse engineering, randomised components (datasets, functions, equations, graphs, networks, matrices ...) and multipart questions.

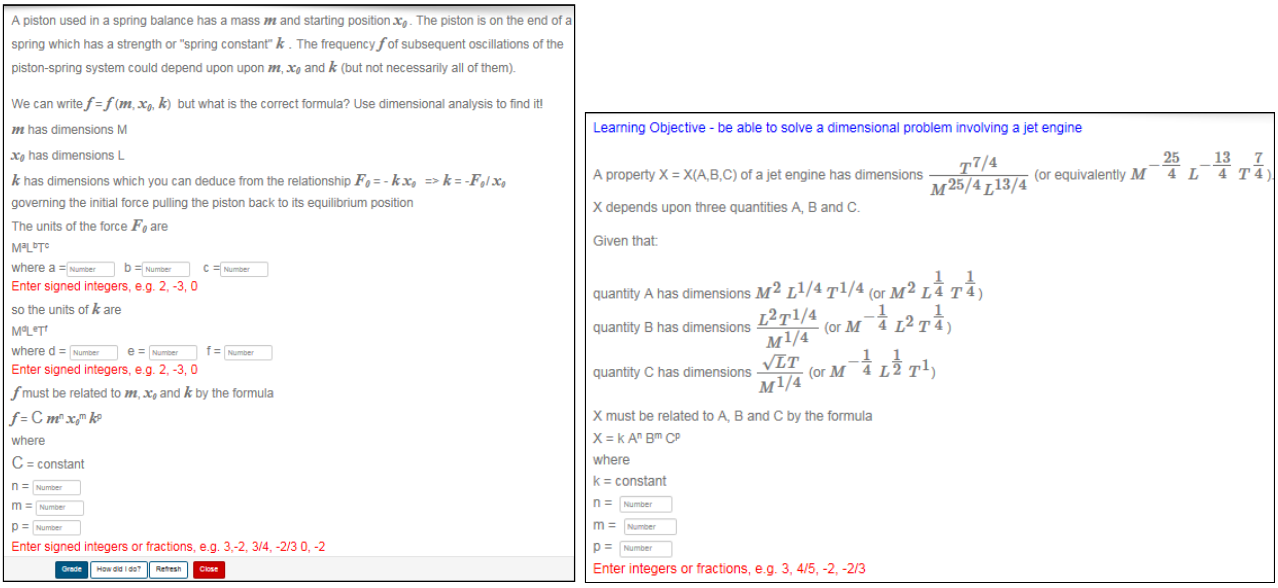


Figure 4. Efficient Question Setting Via Reverse Engineering of Dimensional Analysis

Special Case 1: Dimensional analysis is an extremely useful mathematical technique for solving problems with minimal work, but without a full understanding of the underlying physical processes. It is introduced as part of the Mathematical Models course unit. The left hand screenshot in Figure 4 shows a typical "real-world" question, which leads a student through the solution of a specific problem. The drawback is that finding sufficient realistic dimensional analysis problems to solve and the creation of a question bank is time-consuming. To avoid this, a randomised set of fictitious problems have been developed which allow the technique to be practiced effectively on meaningful questions. To illustrate its implementation, suppose we wish to find an unknown relationship $X = X(A,B,C) = kA^nB^mC^p$. If the dimensions of X, A, B and C are given as $M^{x_1}L^{x_2}T^{x_3}$, $M^{a_1}L^{a_2}T^{a_3}$, $M^{b_1}L^{b_2}T^{b_3}$, $M^{c_1}L^{c_2}T^{c_3}$ respectively, then we deduce that

$$a_1\,n + b_1\,m + c_1\,p = x_1$$
$$a_2\,n + b_2\,m + c_2\,p = x_2$$
$$a_3\,n + b_3\,m + c_3\,p = x_3$$

We could solve for n, m and p using Maple, but cannot easily be assured of user-friendly solutions. Instead the trick is to reverse engineer the question. Rather than setting up a randomised question and solving it, we start with a randomised solution for n, m and p, but then create randomised questions by choosing suitable question parameters. In practice, this simply means randomising a_i, b_i and c_i (i=1..3) and calculating x_1, x_2 and x_3 from the three equations shown above. An important condition, easily implemented in a MapleTA question algorithm, is to ensure that

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

is satisfied. A resulting question is shown on the right hand side of Figure 4. The difficulty of the question is readily adjusted by varying the range of randomised parameters and adding more physical quantities, such as temperature. Reverse engineering is an important technique in setting e-assessment questions in mathematics and allows them to be generated far more simply and reliably than the direct approach of solving a randomly generated problem.

Special Case 2: Algorithms with randomised components lie at the heart of most questions. The generation of simple numerical datasets for fractal box counting is a classic example (Figure 5). A simple logarithmic relationship log N(s) = log C + D log s implies that a graph of log N(s) vs. log s will be a straight line with slope D, the fractal dimension. Data can be generated with or without randomised "noise".

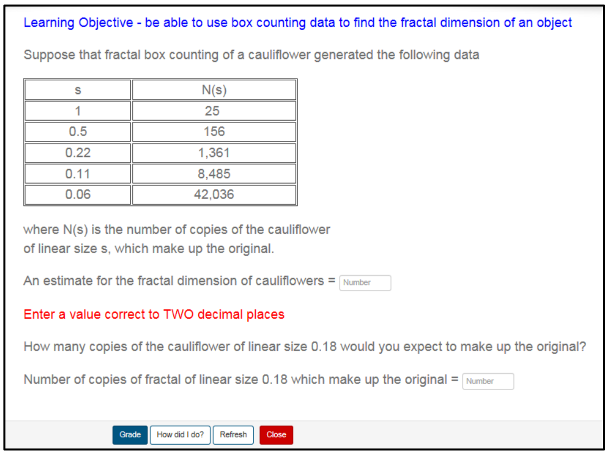


Figure 5. Randomised Datasets for Fractal Box Counting

Large banks of ODE and PDE questions have been developed with randomly generated equations, covering a wide range of types and solution methods.

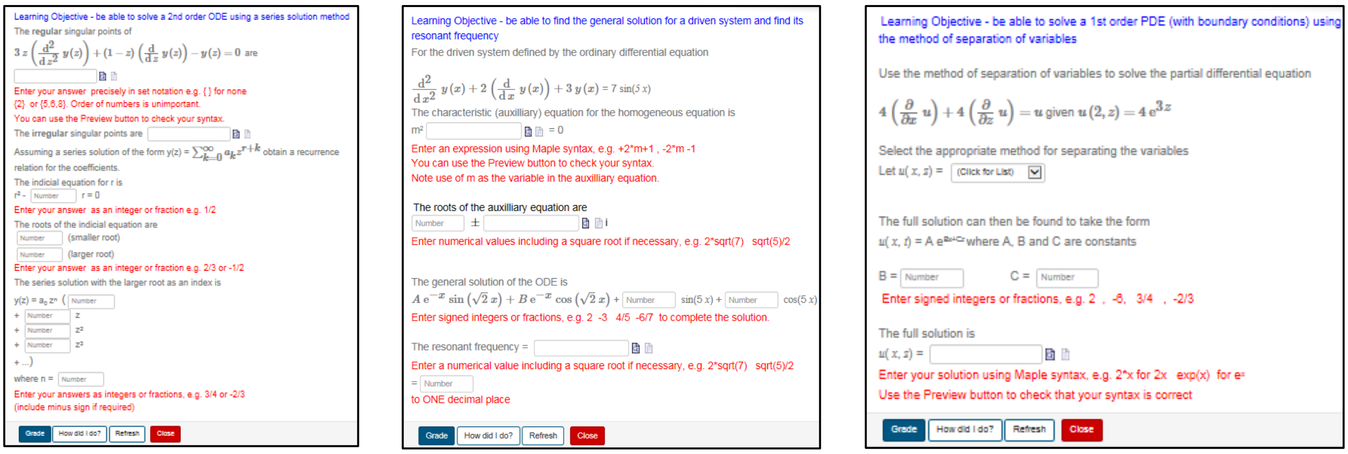


Figure 6. Randomised Equations for ODE and PDE Solution

For these questions, the technique of cloning, i.e. the copying and modification of existing questions, plays an important role in speeding up question production. Often only minor changes are needed to generate a completely different question.

Randomised graphs can be generated in MapleTA very efficiently. The matching question in Figure 7 is an example taken from a linear programming question bank and includes a different, graph for each of the 4 solution possibilities. Any graph that can be generated in Maple can be randomised in MapleTA with minimal effort.

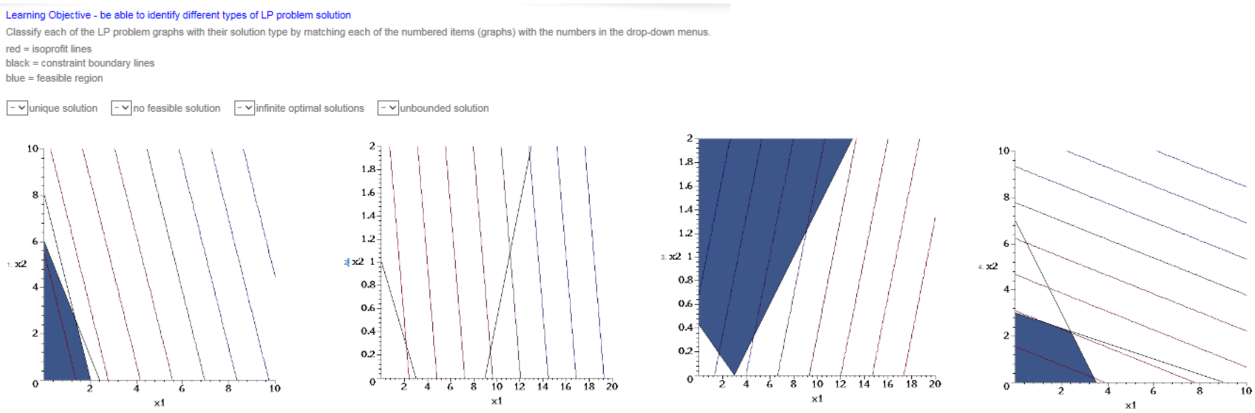


Figure 7. Randomised Graphs for Linear Programming

Other examples of efficient graph plotting, using Maple commands and packages, are shown in Figure 8 below. All the graphs are generated dynamically for each instance of the question with a single command.

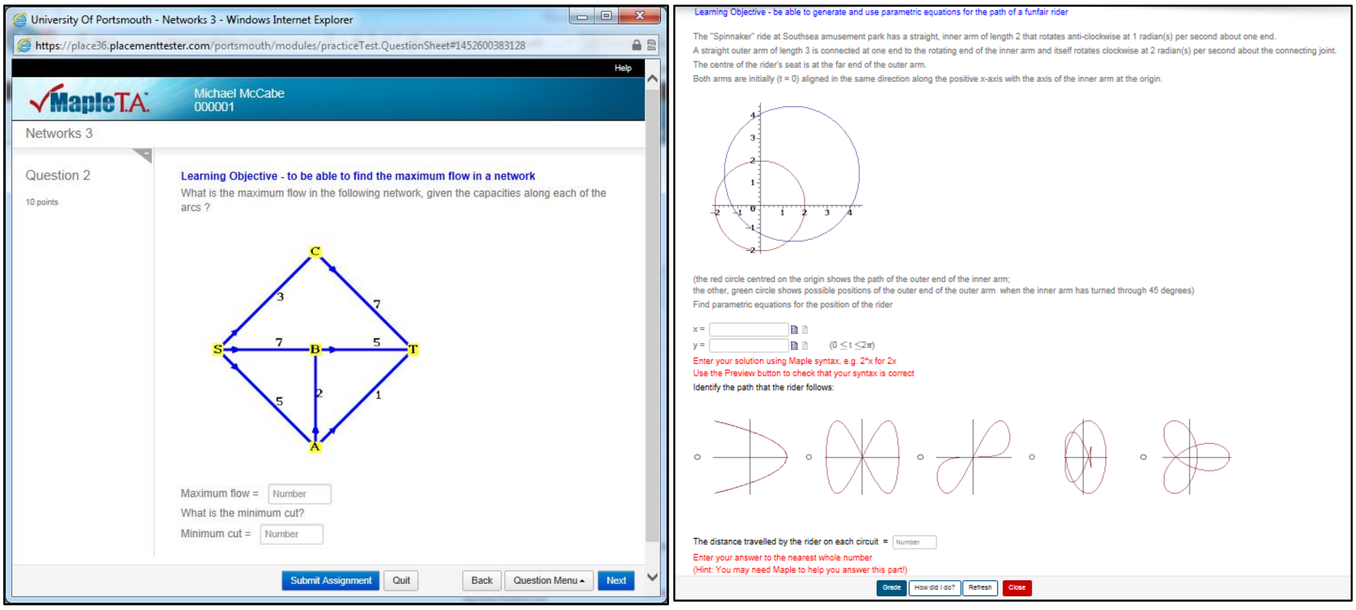


Figure 8. Randomised Graphs for Networks and Parametric Coordinate Problems

Special Case 3: Multi-part questions are used frequently to guide students though common solutions methods. Figure 8 (right) shows the combination of randomised graphs with a multipart question in solving a problem involving parametric coordinates. A further example, shown in Figure 9, is a question which works through statistical hypothesis testing.

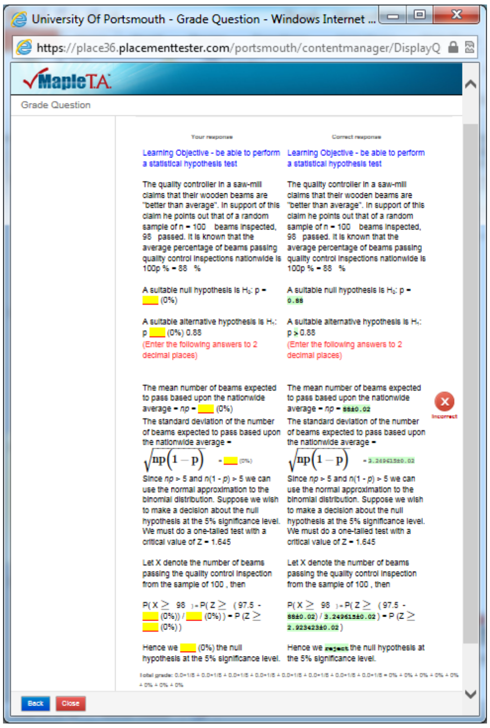


Figure 9. Multi-Part Question for Statistical Hypothesis Testing

Without efficient means of authoring e-assessment questions, their development becomes a slow and unproductive process. The process can become even slower when the time taken to add feedback is taken into account. Experience has shown that reverse engineering, randomised algorithmically generated components and multi-part questions are often the key elements in creating effective, reusable questions.

EFFICIENT AND EFFECTIVE DELIVERY OF FEEDBACK

Basic principles of feedback practice in e-assessment have been well identified over the years, e.g. Nicol and Milligan (2006), but are often extraordinarily difficult to implement. The preparation of detailed, dynamically generated feedback can be extraordinarily time-consuming and may often take longer to produce than the question itself. It is often found to be more efficient to provide traditional written solutions to sample questions. Indeed it can be argued that automated solutions and feedback can eliminate the need to cross-reference different sources of information, such as textbooks, lecture notes and worked examples, making the arena of learning too restricted.

Although students are encouraged to read their lecture notes and worked examples, when they tackle e-assessment questions, some automatically generated feedback is always worthwhile. In MapleTA students can simply ask “How Did I Do?” while they are attempting a multi-part question. For example, when solving an ODE using Laplace transforms (Figure 10), partial solutions can be checked before moving on to later stages of the solution. Thus, mistakes in early parts of the question can be corrected before moving on to the complete solution.

Often a compromise must be reached between time spent on preparing fully automated feedback and more questions for a bank. The pragmatic approach has been to use the “How Did I Do?” option to provide a basic level of feedback without requiring much extra work. When it is routinely available in all questions during weekly practice tests, students usually take full advantage of the help that it gives them.

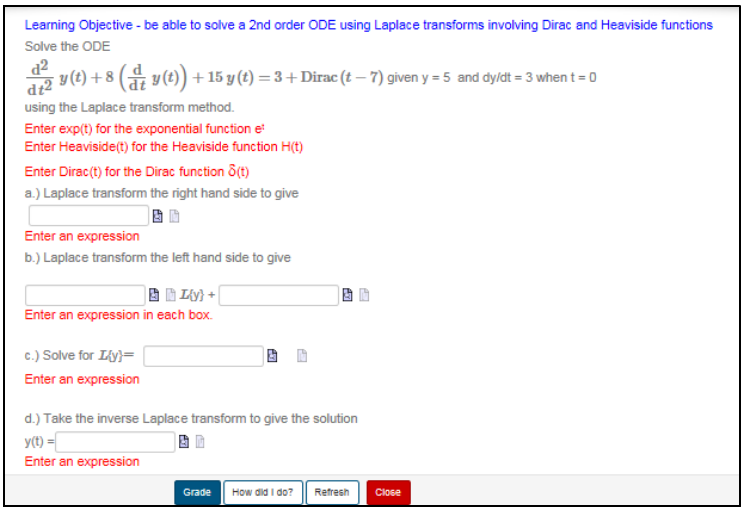
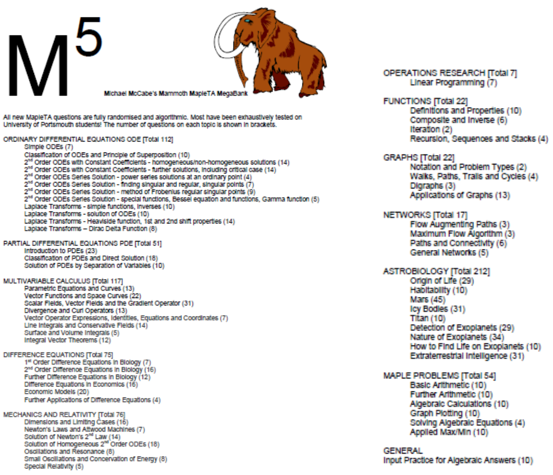


Figure 10. How Did I Do on my Laplace Transform Problem?

THE PRESENT AND FUTURE OF E-ASSESSMENT

Mathematics e-assessment has advanced considerably over a period of more than 25 years, since it was first used at the University of Portsmouth. Software developments have enabled online delivery, CAS checking of responses, randomisation in many different forms, algorithmic question generation, multi-part questions, new question types, targeted feedback and adaptive questions. Changes in the software tools over the first 15 years often made it necessary to abandon existing question banks and write new ones. Maintenance and improvement of question banks is still important, but far less time-consuming than it used to be. For the past 10 years there has been relative stability and the size of question banks has grown (Figure 10) as new topics have been added. With the recent addition of Fourier series, the total number of MapleTA questions now approaches 1000. These are available for MapleTA users of the future, who can also answer the question “How Did I Do?”.


M^5
Michael McCabe's Mammoth MapleTA MegaBank

All new MapleTA questions are fully randomised and algorithmic. Most have been exhaustively tested on University of Portsmouth students! The number of questions on each topic is shown in brackets.

ORDINARY DIFFERENTIAL EQUATIONS ODE [Total 112]
- Simple ODEs (7)
- Classification of ODEs and Principle of Superposition (10)
- 2nd Order ODEs with Constant Coefficients - homogeneous/non-homogeneous solutions (14)
- 2nd Order ODEs with Constant Coefficients - further solutions, including critical case (14)
- 2nd Order ODEs Series Solution - power series solutions at an ordinary point (4)
- 2nd Order ODEs Series Solution - finding singular and regular, singular points (7)
- 2nd Order ODEs Series Solution - method of Frobenius regular singular points (9)
- 2nd Order ODEs Series Solution - special functions, Bessel equation and functions, Gamma function (5)
- Laplace Transforms - simple functions, inverses (10)
- Laplace Transforms - solution of ODEs (10)
- Laplace Transforms - Heaviside function, 1st and 2nd shift properties (14)
- Laplace Transforms – Dirac Delta Function (8)

PARTIAL DIFFERENTIAL EQUATIONS PDE [Total 51]
- Introduction to PDEs (23)
- Classification of PDEs and Direct Solution (18)
- Solution of PDEs by Separation of Variables (10)

MULTIVARIABLE CALCULUS [Total 117]
- Parametric Equations and Curves (13)
- Vector Functions and Space Curves (22)
- Scalar Fields, Vector Fields and the Gradient Operator (31)
- Divergence and Curl Operators (13)
- Vector Operator Expressions, Identities, Equations and Coordinates (7)
- Line Integrals and Conservative Fields (14)
- Surface and Volume Integrals (5)
- Integral Vector Theorems (12)

DIFFERENCE EQUATIONS [Total 75]
- 1st Order Difference Equations in Biology (7)
- 2nd Order Difference Equations in Biology (16)
- Further Difference Equations in Biology (12)
- Difference Equations in Economics (16)
- Economic Models (20)
- Further Applications of Difference Equations (4)

MECHANICS AND RELATIVITY [Total 76]
- Dimensions and Limiting Cases (16)
- Newton's Laws and Attwood Machines (7)
- Solution of Newton's 2nd Law (14)
- Solution of Homogeneous 2nd Order ODEs (18)
- Oscillations and Resonance (8)
- Small Oscillations and Concervation of Energy (8)
- Special Relativity (5)

OPERATIONS RESEARCH [Total 7]
- Linear Programming (7)

FUNCTIONS [Total 22]
- Definitions and Properties (10)
- Composite and Inverse (6)
- Iteration (2)
- Recursion, Sequences and Stacks (4)

GRAPHS [Total 22]
- Notation and Problem Types (2)
- Walks, Paths, Trails and Cycles (4)
- Digraphs (3)
- Applications of Graphs (13)

NETWORKS [Total 17]
- Flow Augmenting Paths (3)
- Maximum Flow Algorithm (3)
- Paths and Connectivity (6)
- General Networks (5)

ASTROBIOLOGY [Total 212]
- Origin of Life (29)
- Habitability (10)
- Mars (45)
- Icy Bodies (31)
- Titan (10)
- Detection of Exoplanets (29)
- Nature of Exoplanets (34)
- How to Find Life on Exoplanets (10)
- Extraterrestrial Intelligence (31)

MAPLE PROBLEMS [Total 54]
- Basic Arithmetic (10)
- Further Arithmetic (10)
- Algebraic Calculations (10)
- Graph Plotting (10)
- Solving Algebraic Equations (4)
- Applied Max/Min (10)

GENERAL
Input Practice for Algebraic Answers (10)

Figure 10. A Megabank of MapleTA Questions

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