HANDWAVER: A GESTURE-BASED VIRTUAL MATHEMATICAL MAKING ENVIRONMENT

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We report on the design and development of HandWaver, *a gesture-based mathematical making environment for use with immersive, room-scale virtual reality. A beta version of* HandWaver *was developed at the* IMRE Lab *at the University of Maine and released in the spring of 2017. Our goal in developing* HandWaver *was to harness the modes of representation and interaction available in virtual environments and use them to create experiences where learners use their hands to make and modify mathematical objects. In what follows, we describe the sandbox construction environment, an experience within* HandWaver *where learners construct geometric figures using a series of gesture-based operators, such as stretching figures to bring them up into higher dimensions, or revolving figures around axes that learners can position by dragging and locking. We describe plans for research and future development.*

Keywords: Geometry, Virtual Reality, Technology

OVERVIEW OF HANDWAVER

HandWaver is a gesture-based virtual mathematical making environment, currently optimized for in-room (as opposed to seated) immersive virtual reality platforms (such as the HTC Vive) that support gesture recognition. From points in space, users can construct uni-, two-, and three-dimensional mathematical objects through iterations of gesture-based operators. Figure 1 shows iterations of the *stretch* operator: a point is stretched into a line segment; the line segment is stretched into a plane figure; the plane figure is stretched into a prism. The hands that are shown in the images are virtual renderings of a user's actual hands, tracked via a *Leap Motion* sensor that is mounted to the virtual reality headset (see Figure 2).

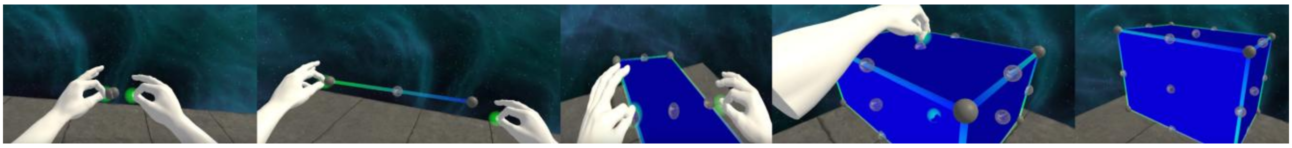

Figure 1. Different cases of the *stretch* operator: a point is stretched into a line segment, the segment is stretched into a plane figure, and the plane figure is stretched into a prism.

Figure 2. A user (red sweatshirt) in the virtual space. The large monitor displays a 2D view of the user's first-person view of the virtual world. The device that tracks the user's hand movements is mounted to the front of the headset he is wearing.

A second gesture-based operator is *revolve.* Users can position an axis in space, select objects to rotate around the axis, and then spin a wheel to revolve the selected objects around the axis. Revolving objects in this way creates surfaces of revolution. Figures 3 and 4 show different cases of the *revolve* operator. In Figure 3, a point is revolved to create a circle; the circle is then revolved around itself to create a sphere; and the circle is revolved around an axis to create a torus.

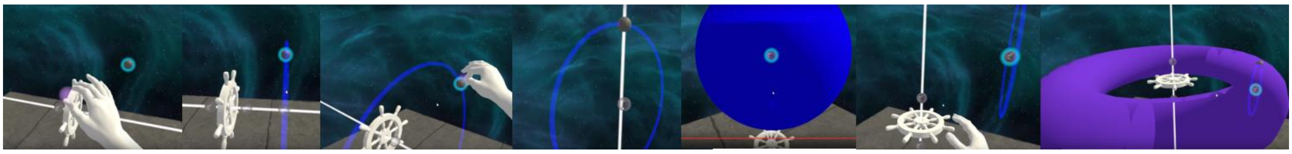

Figure 3. Different cases of the *revolve* operator. The ship's wheel is a spindle that users turn to revolve figures. The line through the ship's wheel is the axis of rotation.

In Figure 4, a segment is revolved parallel to an axis of rotation to create a cylinder; a segment is revolved perpendicular to an axis of rotation to create an annulus; the annulus is revolved around itself to create a sphere with a hole in its center.

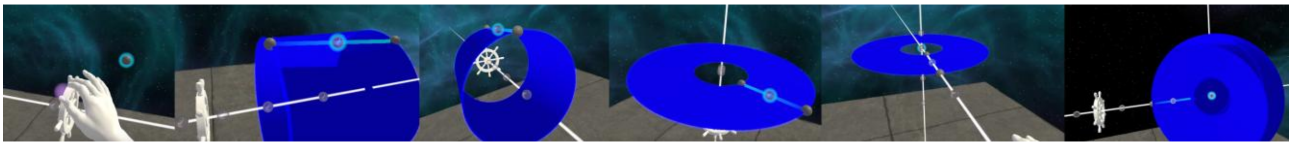

Figure 4. Different cases of revolving a segment. When the segment is parallel to the axis of rotation, the result is a cylinder. When the segment is perpendicular, the result is an annulus. The last image shows an annulus revolved around itself to create a sphere with a hole in its center (note: the hole is visible in the image by slicing the sphere).

We organized the sandbox environment around the *stretch* and *revolve* operators to help learners train their dimensional deconstruction skills (Duval, 2014). Dimensional deconstruction is the process of resolving geometric figures into lower-dimensional components, rather than seeing them as whole, fixed shapes. In the *HandWaver* sandbox, learners can fluidly move from lower-dimensional shapes (e.g., circles) to their higher dimensional analogs (e.g., spheres) and vice versa. The environment brings plane and solid geometry together—subjects that have been separated from each other in the usual presentation of geometry in K-12 schools.

The solid analogs of plane figures, in particular sphere-and-plane constructions, are "seldom developed" or "slighted...owing to their theoretic nature" (Franklin, 1919, p. 147). Three-dimensional dynamic geometry software (e.g., GeoGebra or Cabri 3D) has made it possible to engage in such constructions, however the limitations of two-dimensional screens has constrained their practicability. But for users immersed in a three-dimensional space—where the user has natural control over the angle at which an object is viewed, is able to move and manipulate the object in space, and can readily select the components of a figure to be incorporated into a new construction—three-dimensional constructive geometry becomes more feasible.

Thus, a final feature of the sandbox environment is three-dimensional analogs of classic construction tools. The *arctus* tool (Figure 5) allows users to make a sphere centered at a point, through any other point. The size of the arc shown in the figure is variable, and the midpoint of the *arctus* tool can be locked to any point in the display. *Arctus* is a spatial compass that creates spheres. The user sets the arc to have the desired radius and then generates a sphere by spinning the arc through space.

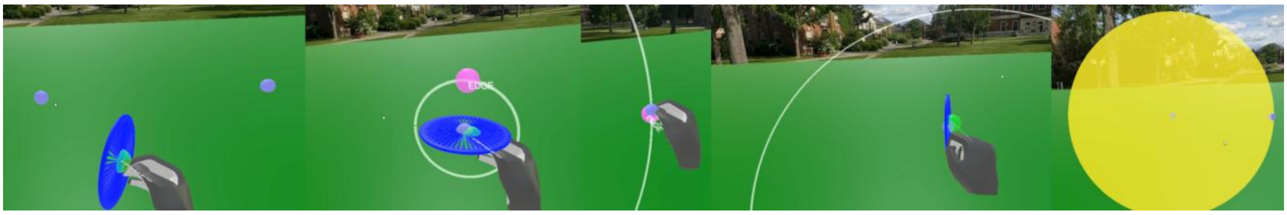

Figure 5. The *arctus* tool being used to inscribe a sphere. Users position the tool on a center point and on a point on the surface of the sphere . To generate the sphere, one turns the circle through space by spinning the blue wheel.

The *flatface* tool (Figure 6) allows users to define a plane through any three points. A user sets one of the lines of the *flatface* to coincide with two of the three points. Once in place, the user sets the second line so that it is collinear with the third point. To generate the plane, one acts with the *stretch* gesture on one of the lines of the flatface. We implemented plane-and-sphere constructions via gesture- (and motion-) based virtual tools to mimic the physical actions of spinning a compass or drawing a line with a straightedge. Our goal in doing so was to highlight the manual history of making geometric figures.

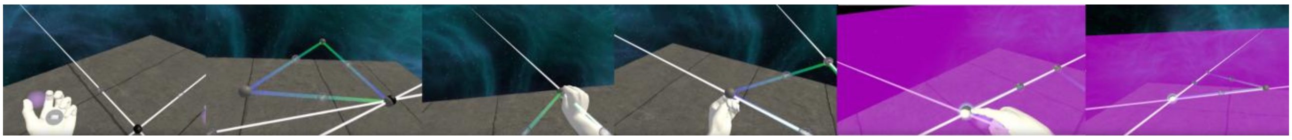

Figure 6. Series of images showing the *flatface* tool being used to spawn a plane.

With *arctus* and *flatface*, learners can complete solid geometry construction tasks that are inherently virtual, such as constructing a tetrahedron from three spheres (see Figure 7).

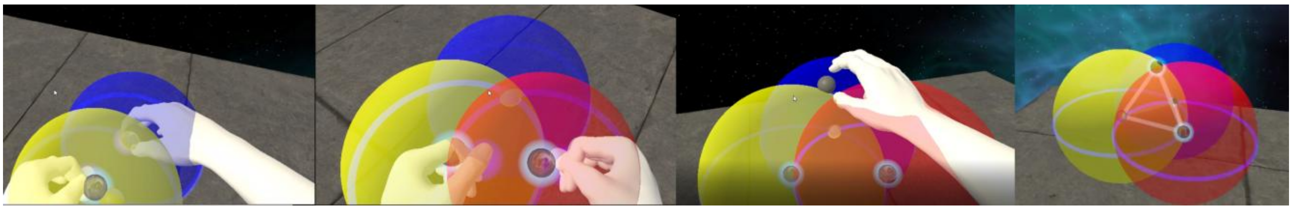

Figure 7. Constructing a tetrahedron from three-spheres in the *HandWaver* sandbox.

These tools provide an occasion for learners to explore how plane geometry construction protocols can be extended to higher dimensions. Other experiences within the *HandWaver* environment include a volume lab, an operator lab, and *LatticeLand*, which is a spatial analog of the geoboard

(Kennedy & McDowell, 1998). Users can define the edges or faces of polyhedra by selecting a circuit of lattice points with a virtual pin (see Figure 8).

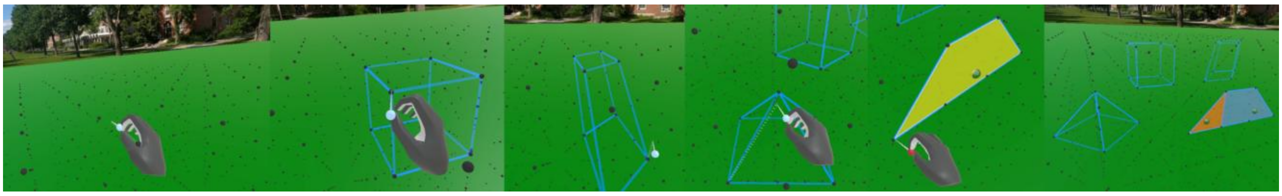

Figure 8. Connecting the dots in *LatticeLand* to define a the edges of a cube (second frame), a parallelepiped (third frame), a pyramid (fourth frame), and a trapezoid (fifth frame); the sixth frame shows the trapezoid cut into components (the orange triangle, the blue trapezoid).

MOTIVATION AND DESIGN CONSIDERATIONS

Our primary goal in developing *HandWaver* was to provide a space where learners at all levels could use their hands to act directly on mathematical objects, without the need to mediate intuitions through equations, symbol systems, keyboards, or mouse clicks (Sinclair, 2014). We designed the environment around natural movements of user's hands to foreground the connection between diagrams and gestures (de Freitas & Sinclair, 2012; Chen & Herbst, 2013). As one example of how the environment realizes this connection, the *stretch* operator multiplies (Davis, 2015) single points into many to form a segment, or multiples single segments into many to form a plane figure, or multiplies a single plane figure into many to form a solid. The notion that *n*-dimensional figures consist of adjoined (*n*-1)-dimensional figures is foregrounded by the generative use of the stretching gesture.

Gestural interfaces (Zuckerman & Gal-Oz, 2013), where objects can be manipulated in natural, intuitive ways by movements of one's hands, allow a degree of direct access to virtual objects that have been shown to facilitate learning (Abrahamson & Sánchez–García, 2016) while minimizing cognitive barriers (Sinclair & Bruce, 2015; Barrett, Stull, Hsu, Hegarty, 2015). Virtual environments with gestural interfaces have affordances for translating multimodal cues—e.g., head or hand movements—into mathematical operations, such as projecting a plane figure into three dimensions by pulling it up into space. The name of the environment, *HandWaver*, is an attempt to reposition "hand waving"—a term used to criticize mathematics that is insufficiently rigorous—as a means for doing mathematical work.

A further motivation for developing a construction environment with a gesture-based interface was to make it accessible to younger learners. Soon, children will routinely and increasingly incorporate virtual reality environments into their leisure activities. They will be playing games that require spatial reasoning and problem solving skills—imagine, for example, an immersive first-person version of *Monument Valley* (Ustwo, 2014)—but what will they be doing in schools?

Currently, children's encounters with geometry in elementary schools are limited to shape recognition and naming tasks (Bruce & Hawes, 2015). Yet a growing body of research indicates that children have the interest and capacity to train their spatial reasoning skills (Hallowell, Okomato, Romo, La Joy, 2015; Whiteley, Sinclair, & Davis, 2015; Taylor & Hutton, 2013) and study meaningful mathematics (Newton & Alexander, 2013; Sinclair & Bruce, 2015) from the moment they enter the schoolroom door. New modes of interacting with virtual mathematical objects (Hwang & Hu, 2013; Kaufman 2011) have the potential to expand children's access to deep geometric ideas. For all of its educational promise, however, virtual reality is on a track to follow the slow, complex process of technology acceptance and adoption that is standard in schools and that falls short of true integration (Ertmer, 1999; Inan & Lowther, 2010). Given how difficult it has been, historically, to incorporate promising technologies into classrooms at scale, there is every reason to believe that the educational potential of virtual reality will remain unfulfilled.

Our final reason for developing *HandWaver* is thus perhaps the most important: We developed the environment so that we would be able to critically investigate the disparity between what is and what could be in using virtual reality to enhance mathematics education. There is a "scarcity of bold research on interactive mathematics learning" that "impedes the formulation of empirically based progressive policies concerning the integration of technological environments into educational institutions" (Abrahamson & Sánchez–García, 2016, p. 204). In addition to investigating how students explore mathematical structures within an immersive virtual mathematics laboratory (Bock & Dimmel, *in press*), we are convening study groups to investigate (1) how practicing teachers would manage the challenges and opportunities of incorporating virtual reality technology into their instruction, and (2) how pre-service teachers could be adequately prepared for teaching with such technology.

DEVELOPMENT PROCESS

The environment is built in room-scale virtual reality, with a 4 meters by 8 meters activity space. This provides affordances of consistent head tracking and perspectives from varied physical heights and locations, which are not available with seated virtual reality (e.g., GearVR) and 360-video hardware (e.g., Google Cardboard). Recent advances in hardware have made significant improvements in performance and cost. The HTC Vive and Oculus Touch head mounted devices (HMDs) both provide room-scale virtual reality with consumer-grade hardware and cost similar to other classroom technology (e.g., Interactive White Boards). We chose the HTC Vive for it's larger activity space, early room-scale availability and local multiplayer in a shared activity space. Recent advances in consumer GPUs have expanded access to the processing power required to drive these HMDs to consumer workstations. The combination of improvements in processing and in the HMDs has minimized previous issues with motion sickness. Room-scale optimizes problems with posture and fatigue in the environment, and also allows for more advanced image processing to improve immersion. Finally, the LeapMotion Orion SDK allows for reliable hand tracking integrated across the HTC and Oculus platforms.

RESEARCH PLANS

We are engaged in parallel lines of research using *HandWaver*. One line of research concerns documenting student encounters with mathematical objects in the virtual space. The immersive nature of the environment, combined with the gestural interface, provides a level of control over perspective, orientation, and position relative to mathematical objects that is difficult to replicate with other display technologies. Even the relatively straightforward means for rotating the graphics view in the 3D version of GeoGebra is complicated when compared to moving one's head, walking around a figure, or examining it from several different angles in quick succession. How do students use the angle of their gaze, the position of their bodies relative to virtual mathematical figures, or the ability to quickly change the scale of figures—from something that one could hold in one's hands, to something that one could fit inside—to explore mathematical structures?

This line of research frames activity within *HandWaver* (e.g., the volume laboratory) using the *conceptions-knowing-concept* (cK¢) model of conceptions (Balacheff & Gaudin, 2010; Balacheff, 2013): the virtual environment creates a *milieu* where students encounter problems that they explore using a suite of virtual operators, such as the ability to compare solid figures by superposition (see Figure 9).

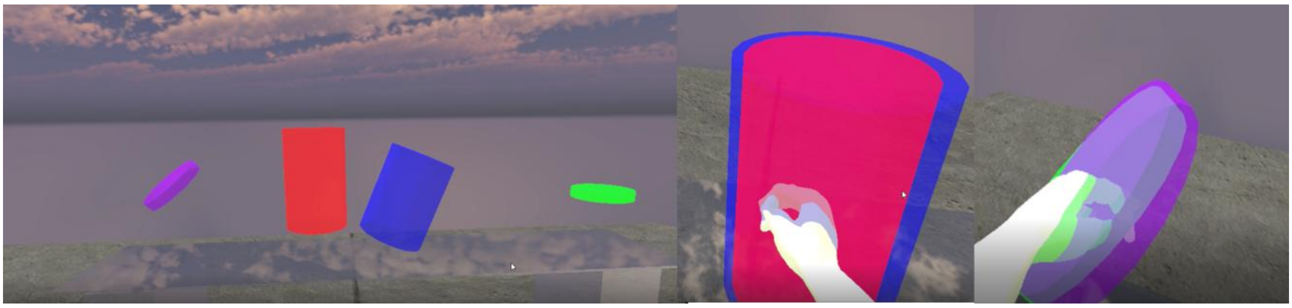

Figure 9. Comparing volumes of virtual solids by superpositioning.

In one study (Bock & Dimmel, *in press*), we used semi-structured interviews where participants—three master's students pursuing certification as science teachers—were asked to think-aloud as they explored the volume of a pyramid. One of the operators available to participants was the ability to dynamically change the pyramid by pinching and dragging its apex in space. Participants could lock the apex in the *z*-direction (shearing) or *xy*-directions (elongating) to control how the apex moved. Other operators included the ability to enclose the pyramid in a unit cube and add additional pyramids to it (see Figure 10).

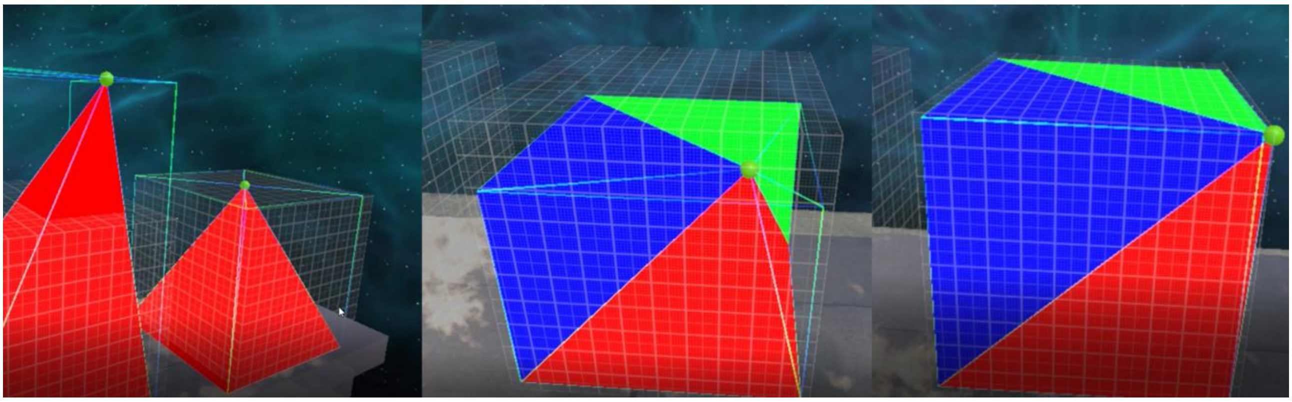

Figure 10. Enclosing the pyramids in a unit cube, adding additional pyramids, and adjusting the pyramids by moving the apex.

Participants could then explore how the volumes of the added pyramids were affected by movements of the apex. One strategy used by participants in this study was to reason about the volume of a pyramid by analyzing how the surface area of its faces was affected when the apex was moved in different ways. We are planning an interview study that would investigate how participants use the gesture-based operators available in the sandbox to construct different geometric figures.

A parallel line of research pertains to issues of instructional implementation: How do practicing and preservice teachers imagine incorporating virtual reality technology into their teaching? What support do they need? What barriers do they anticipate? For this research, we are developing multiplayer and partial immersion modes so that *HandWaver* could be used by a teacher with a whole classroom. The multiplayer mode will allow more than one user to be in the same virtual world at one time. The partial immersion mode will allow other users to view what is happening in the virtual world through a tablet. The partially immersed users will also be able to have some limited interactions with the virtual world, such as using gestures to control their angle of view, their position within the environment, or to construct figures. We are anticipating a time in the not-too-distant future when it will become feasible for a classroom to have multiple VR consoles that will allow students to work on problems in groups. In such configurations of virtual reality enhanced mathematical explorations—what we call *virtual mathematics laboratory experiences*—some students would be fully immersed in a virtual world and others would access the environment via a gesture-tracking tablet. We have a dedicated laboratory classroom space at the University of Maine where we will convene groups of teachers to study the instructional potential of teaching in a virtual reality-enabled classroom. Groups of participating teachers will explore and critique the *HandWaver* environment. They will work with each other to devise plans for how such an environment could be used in their teaching and anticipate obstacles they would expect to encounter. The first study group will be convened during the 2017-2018 academic year.

FUTURE DEVELOPMENT

The development of *HandWaver* is ongoing. We are planning a second release that will have new experiences, new modes of interacting within the environment, and new tools for use within existing experiences. A new experience that we are developing is a spherical trigonometry and nautical science lab, where users would be able to investigate properties of triangles that are inscribed on the surfaces of spheres. We are also developing a suite of measurement tools for use in the sandbox and volume labs, such as a paint roller that has different shaped heads (e.g., triangular, rectilinear, circular) that can be varied in size. Users would be able to “roll on” various area units to cover plane figures. The purpose of such a tool would be to provide a visual representation that units for measuring area are two-dimensional.

The advent of consumer grade virtual reality consoles (e.g., Oculus, HTC Vive) is likely to usher a frenzy of development of commercial, virtual reality educational content. If such development follows the path of educational apps, a preponderance of the mathematics education content that is developed for virtual reality consoles will amount to little more than immersive, visually engaging flashcards (Davis, 2015). By designing and developing the *HandWaver* environment, we are attempting to ensure that research-based ideas about the nature of productive mathematical activity are represented in this next generation of virtual learning environments.

REFERENCES

Abrahamson, D., & Sánchez–García, R. (2016). Learning is Moving in New Ways: The Ecological Dynamics of Mathematics Education. *Journal of the Learning Sciences*, (online first edition).

Barrett, T. J., Stull, A. T., Hsu, T. M., & Hegarty, M. (2015). Constrained interactivity for relating multiple representations in science: When virtual is better than real. *Computers & Education*, *81*, 69-81.

Bruce, C. D., & Hawes, Z. (2015). The role of 2D and 3D mental rotation in mathematics for young children: what is it? Why does it matter? And what can we do about it?. *ZDM*, *47*(3), 331-343.

de Freitas, E., & Sinclair, N. (2012). Diagram, gesture, agency: Theorizing embodiment in the mathematics classroom. *Educational Studies in Mathematics*, *80*(1-2), 133-152.

Chen, C. L., & Herbst, P. (2013). The interplay among gestures, discourse, and diagrams in students’ geometrical reasoning. *Educational Studies in Mathematics*, *83*(2), 285-307.

Davis, B. (2015). Gumm(i)ing up the works? Lessons learned through designing a research-based “app-tutor”. *Proceedings of the 12th International Conference on Technology if Mathematics Teaching*, Faro, Portugal.

Dimmel, J. K., & Herbst, P. G. (2015). The Semiotic Structure of Geometry Diagrams: How Textbook Diagrams Convey Meaning. *Journal for Research in Mathematics Education*, *46*(2), 147-195.

Duval, R. (2014). Commentary: Linking epistemology and semio-cognitive modeling in visualization. *ZDM*, *46*(1), 159-170.

Ertmer, P. A. (1999). Addressing first-and second-order barriers to change: Strategies for technology integration. *Educational Technology Research and Development*, *47*(4), 47-61.

Franklin, P. (1919). Some Geometrical Relations of the Plane, Sphere, and Tetrahedron. *The American Mathematical Monthly*, *26*(4), 146-151.

Hallowell, D. A., Okamoto, Y., Romo, L. F., & La Joy, J. R. (2015). First-graders’ spatial-mathematical reasoning about plane and solid shapes and their representations. *ZDM*, *47*(3), 363-375.

Hwang, W. Y., & Hu, S. S. (2013). Analysis of peer learning behaviors using multiple representations in virtual reality and their impacts on geometry problem solving. *Computers & Education*, *62*, 308-319.

Inan, F. A., & Lowther, D. L. (2010). Factors affecting technology integration in K-12 classrooms: A path model. *Educational Technology Research and Development*, *58*(2), 137-154.

Kennedy, J., & McDowell, E. (1998). Geoboard Quadrilaterals. *The Mathematics Teacher, 91*(4), 288-290.

Kaufmann, H. (2011). Virtual Environments for Mathematics and Geometry Education. *Themes in Science and Technology Education*, *2*(1-2), 131-152.

Moss, J., Bruce, C. D., & Bobis, J. (2015). 5 Young Children’s Access to Powerful Mathematics Ideas. *Handbook of International Research in Mathematics Education*, 153.

Murphy, C. (2012). The role of subject knowledge in primary prospective teachers’ approaches to teaching the topic of area. *Journal of Mathematics Teacher Education*, *15*(3), 187-206.

Newton, K. J., & Alexander, P. A. (2013). Early Mathematics Learning in Perspective: Eras and Forces of Change. In *Reconceptualizing Early Mathematics Learning* (pp. 5-28). Springer Netherlands.

Sinclair, N. (2014). “Generations of research on new technologies in mathematics education.” *Teaching mathematics and its applications*, *3*, 166-178.

Sinclair, N., & Bruce, C. D. (2015). New opportunities in geometry education at the primary school. *ZDM*, *47*(3), 319-329.

Taylor, H. A., & Hutton, A. (2013). Think3d!: training spatial thinking fundamental to STEM education. *Cognition and Instruction*, *31*(4), 434-455.

Ustwo (2014). Monument Valley [computer game]. Available at: https://www.monumentvalleygame.com/mv1

Whiteley, W., Sinclair, N., & Davis, B. (2015). What is spatial reasoning? In Davis, B. (ed.) *Spatial reasoning in the early years: Principle, assertions, and speculations.* New York, NY: Routledge.

Zuckerman, O., & Gal-Oz, A. (2013). To TUI or not to TUI: Evaluating performance and preference in tangible vs. graphical user interfaces. *International Journal of Human-Computer Studies*, *71*(7), 803-820.