MAKING GOOD PRACTICE COMMON PRACTICE BY USING COMPUTER AIDED FORMATIVE ASSESSMENT

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Rich technological environments present many opportunities for guided inquiry in the mathematics classroom. In this paper we focus on the role of the teacher supporting the forming and proving of conjectures by the students, during a whole class discussion. We examine the practices of an expert teacher that conducts a classroom discussion based on students' conjectures formed while working in pairs with a dynamic geometry environment (DGE). Specifically, we analyse the way the teacher categorizes the different conjectures, and then addresses them during the whole class discussion. We suggest that this categorization could be offloaded onto a technological platform that would do it automatically, thus making this type of information accessible not only to teachers that could perform this categorization on the spot.

Keywords: Instrumental orchestration, classroom discussions, conjectures, Dynamic Geometry Environment (DGE)

INTRODUCTION

Defining, analysing, and trying to distribute good practice of teachers in the mathematics classroom is an ongoing challenge for the research community (Chazan & Ball, 1999). Guided inquiry tasks are open ended tasks that usually have more than one solution, and often require taking into account various dimensions that were not addressed in previous learning, thus requiring the students to go through a problem-solving process. Promoting and evaluating this process presents challenges for teachers. In the case of computer based guided inquiry, where students are expected to form and reason about conjectures, the primary role of the teacher is to promote and organize discussions (Yerushalmy & Elikan, 2010).

Orchestrating the work of students in a technological environment, referred to by Trouche (2004) as instrumental orchestration, while gathering information about students that could be used for formative assessment, presents challenges for the teachers (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010). In terms of evaluation, formative assessment requires the teacher to draw on information from teaching as feedback to modify accordingly the teaching of the students the information was gathered about (Black & William, 1998). The abundance of data that is created and could be analysed when students engage in rich inquiry tasks on a technological platform presents a challenge for teachers. Some researchers suggest the use of technological platforms for the gathering and display of student answers (Arzarello & Robutti, 2010; Clark-wilson, 2010). Another practice observed by Panero & Aldon (2015) was the combination of automatically collected digital data with tradition pencil and paper work that was used by the teacher in formative assessment in real-time. Another strategy suggested by Olsher, Yerushalmy, and Chazan (2016) would be to offload some of the processing of the data onto a digital platform, automatically categorizing student answers by mathematical characteristics, thus enabling the teacher to have accessible processed data to inform his decision making.

Yet, although data is accessible, and practices are studied, guided inquiry is not a prominent practice in mathematics classroom. One way to address is to explore ways to study and promote *good practice* of teachers in technologically rich environments that present students with guided inquiry.

CONTEXT OF THE STUDY

This study recalls a recorded guided inquiry session with 24 students from grades 9th-10th, working in pairs. The students are using a first generation DGE (Geometric Supposer), which served as a technological platform used to elicit conjectures for about half a year prior to the recorded lesson. The lesson's was planned to summarize major theorems of similarity in triangles. The students are walked through the construction on the board, while the teacher describes the actions, and the students get a printed version of the task as well.

The leading research aim is answering the question whether it is possible to identify good practices about conducting conjecture based discussions in the classroom, and whether the categorization of conjectures in a way that facilitates these practices by providing these categorizations automatically.

METHODOLOGY

In order to address the research question we study a classroom in which the teacher needs to gather, process, and utilize information that is generated by his students while conducting an individual inquiry activity using a DGE. Truoche (2004) uses the term "instrumental orchestration" to describe didactic configurations and the way that they are being exploited in the classroom, and also suggests them as a construct that could "give birth to new instrument systems" (ibid, p.304). In the observed lesson this framework is suitable to describe the way the teacher works with the students answers, and suggest "new instrument systems" whether available within the given environment or supported by different technological platforms.

For the analysis presented in this paper, we have analyzed a recording of a one-hour lesson in a classroom, which serves as our main data source. In addition we draw upon the design principles of the STEP platform for use in classrooms that are equipped with personal digital devices.

In the next part, we describe the task presented to the students. Following that part we analyse the way the teacher orchestrates the discussion surrounding the conjectures raised by the students. Specifically, we analyze the categorization of the conjectures in terms of placement on the blackboard (if at all), and type of treatment they are given by the teacher (i. e. acknowledging the difficulty to prove a certain conjecture, or specifying the underlying constraints). As this orchestration requires a lot of real-time decision making by the teacher, we then examine how the use of automatic analysis tools (e. g. the STEP platform) could offload some of this orchestration, creating new instrument systems thus possibly making this process more accessible for other teachers.

The task

Construct an acute triangle. Draw the altitudes from each one of the triangle points, and mark the feet of the altitudes D, E, F. Label the intersection point G. reflect point G over each side of the triangle. What’s the relationship between triangle DEF which is formed by connecting the feet of each altitude, and the triangle formed by connecting the image points of G, the original triangle, and angles, segments? Investigate anything that you can find. Write out formal conjectures as we have been doing in class.

In figure 1 appears a sketch that resembles the one that was drawn on the blackboard in the recorded lesson.

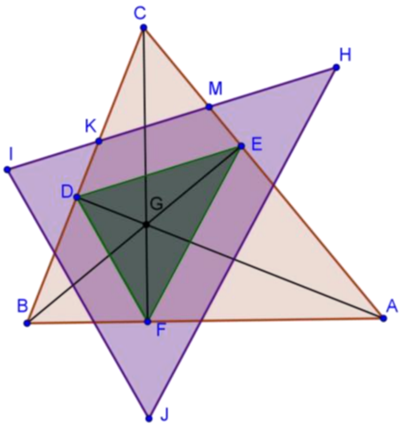

Figure 1. A sketch of the geometric construction discussed in the classroom

DATA ANALYSIS

During the recorder session, we have identified 11 conjectures that were addressed in the classroom (Table 1). Once the seven conjectures were listed, the teacher initiated a discussion aiming to review the conjectures and the argumentation and underlying supposing at the base of each conjecture.

Listed on the left side of the blackboard	Listed on the right side of the blackboard	Conjectures that were raised by students but did not appear on the black board
A1. $\Delta IHJ \sim \Delta DEF$	B1. If ΔABC is isosceles and acute then: $\measuredangle ACB$ is geometric mean of $\measuredangle FED \;\; \measuredangle FDE$	C1. $\frac{Sides\ of\ \Delta\ IHJ}{Sides\ of\ \Delta DEF} = 2$
A2. $\frac{Area\ of\ \Delta\ IHJ}{Area\ of\ \Delta DEF} = 4$	B2. Bisector is the same as altitude. $\overline{BE},$ bisects $\measuredangle DEF$ and $\overline{BE}\ extended$ bisects $\measuredangle IHJ$	C2. Corresponding sides are parallel
A3. $\frac{Perimeter\ of\ \Delta\ IHJ}{Perimeter\ of\ \Delta DEF} = 2$	B3 $\measuredangle ACB \cong \measuredangle FED \cong \measuredangle FDE$ (ΔABC is isosceles)	C3. $\Delta JHI \sim \Delta DEF$ bisector of H also bisects E.
A4. $\overline{IH} \parallel \overline{DE}, \overline{EF} \parallel \overline{HJ}, \overline{DF} \parallel \overline{IJ}$		C4. <u>IF ABC is isosceles:</u> Creates two other isosceles triangles.

Table 1. Conjectures raised by students according to their appearance on the black board

When addressing these conjectures, we have identified four strategies used by the teacher. The first strategy can be demonstrated with conjecture A1 (table 1), was to state the conjecture on the left side of the blackboard, and then ask how many of the students agree with the conjecture:

Teacher: You think that triangle IHJ is similar to triangle DEF [writes $\Delta IHJ \sim \Delta DEF$ on the blackboard]. Raise your hand if you believe that's true? [All of the students raise their hand] oh. So everyone does. Great.

The second strategy can be demonstrated using conjectures C1 and A3, was not to write the initial conjecture, but to either refine it by himself (C3 turned into B2) or by involving the students, as can be shown from the following excerpt:

Student 1: Their sides are two to one.

Teacher: The ratio of their sides is two to one.

Students: Perimeter.

Teacher: Perimeter is two to one [writes $\frac{Perimeter\ of\ \Delta\ IHJ}{Perimeter\ of\ \Delta DEF} = 2$ on the blackboard]. The perimeter of triangle IHJ, to the perimeter of triangle DEF is two. Which means the ratio of their sides is also two to one.

The third strategy can be demonstrated using conjecture B1, as to write the conjecture with the additional constraints relevant to it on the right hand side of the board (on the right side of the sketch). In this case the teacher also assigns ownership of this conjecture and the additional constraints to the students that raised it:

Student 2: If triangle ABC is isosceles, then hmm, the measure of the angle ACB equals either of the two base angles in the two smaller triangles. Because those two smaller triangles are also isosceles.

Teacher: You and Jenifer worked a lot with isosceles triangles, didn't you? [Drawing on the blackboard a sketch represented in Figure 2] OK what do you claim?

Student 2: That angle ACB equals, is congruent to angle FED and angle FDE

Teacher: [writes $\measuredangle ACB \cong \measuredangle FED \cong \measuredangle FDE$ (ΔABC is isosceles) on the blackboard].

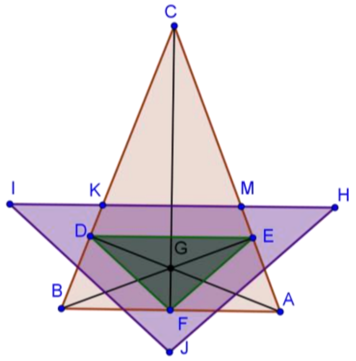

Figure 2. A sketch of the constrained case students addressed in the classroom

The fourth strategy used was in cases that were stated on the right side of the blackboard, but were general. There was one occurrence of a case such as this, conjecture C3 that was raised and then processed to conjecture B2 that was finally written on the board. This conjecture was not one that the lesson plan for this activity prepared the teacher for.

Table 1 suggests rough categories for the conjectures as they were addressed by the teacher. The Conjectures that appeared on the left side of the blackboard (A1-A4) were conjectures that the teacher expected, and went over their justifications in class. The conjectures that did not reach the blackboard (C1-C3) required some additional rephrasing or generalization in order for them to be well defined and represented, and their evolved form eventually appeared on the board. The conjectures that appeared on the right side of the blackboard were either case specific conjectures (B1, B3), or conjectures that were more advanced compared with the learned content.

The teacher then moves on with a reflection, sharing his thoughts and his planning with the students. In this the teacher, a well-established authority figure in the classroom, demonstrated that he was not completely prepared for everything that appeared - on the contrary. He was happy to be surprised by the students' ideas he did not expect. He asks which conjecture the students thought surprised him, and they stated the bisector one (B2), to which the teacher agreed. He then states that he will not address all of the conjectures, but he will do the ones on the left side, stating that these are the ones that everyone found; he then goes over the proofs for all of them. Then he turns to the right hand side of the board, and categorises the conjectures further: conjectures B1 and B3 are referring to the sketch in Figure 2, and are given as homework, but conjecture B2 is referred to as a general conjecture, true for any triangle. The teacher states it might be difficult for them to prove, and gives them additional time and offers hints if they will have difficulties. So the reflection about the "surprising" aspect in students' conjectures serves beyond the issue of authority; this could be the teachers' way to categorize conjectures as being more or less trivial (expected) to be proved.

DISCUSSION

Expert teachers have the skills and knowledge to filter and categorize student answers during the classroom session even in complex situations of inquiry based learning. Yet, this ability is not common practice, especially when gathering information from technologically based platforms (Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010). As also appears in the case presented above, the teacher refers to cases that were not expected by him as cases that he might not address in the classroom. Olsher, Yerushalmy, & Chazan (2016) suggest the use of automatic filtering of student responses to make this type of information more accessible for teacher use as means for formative assessment. One example that is suggested by Olsher et al. (2016) is the STEP platform, which enables teachers to predefine mathematical properties of student answers, for the platform to automatically analyze and categorize for increasing the accessibility of the teacher to the student answers.

For the case presented, the categorization of the teacher could be mapped into an automatic filtering scheme. As the topic of this lesson is similarity, many conjectures that address certain characteristics of similarity are expected to be raised: ratio between sides, areas, relationship between corresponding segments (e.g. parallel segments). Even student mistakes that are prominent in the teaching of the similarity could be expected (e.g. mistaking between the ratio of segments and the ratio between areas).

These relations could be predefined and automatically recognized by the platform (e.g. STEP), making relevant data such as: is this relation addressed by the students? If so, by how many of the students? Furthermore, as DGE's are currently even more flexible than the Geometric Supposer in terms of the student's ability to drag pre-constructed figures, categorizing by the mathematical properties stated could be even more important as the students' example spaces potentially grow even wider. One example for filtering student answers is by determining whether they added constraints to the given situation, and by that potentially limited the generality of their answer, such as was demonstrated in conjectures B1 and B3 that were related to isosceles triangles. By defining the expected relations, we are also setting the stage for the unexpected relations to appear. They could easily be addressed by the teacher, and also automatically determine their correctness. By acknowledging that the platform will not identify the entire space of relations that students raised we leave room for student creativity, which is a substantial part of inquiry based activities, but also might keep educators from using automatic assessment platforms. In later sessions teachers could choose to incorporate these relations into the detected relations scheme if they see it fit.

We conclude in suggesting that the automatization of the categorizing and surveying of the student answers, beyond their correctness, could serve as a tool for teachers in their instrumental orchestration of a technologically based guided inquiry learning environment.

REFERENCES

Arzarello, F., & Robutti, O., (2010). Multimodality in multi-representational environments. *ZDM, 42(7)*, 715-731.

Black, P., & Wiliam, D., (1998), *Assessment and Classroom Learning*, Assessment in Education, Vol. 5, 7-74.

Chazan, D., & Ball, D.(1999). Beyond being told not to tell. *For the Learning of Mathematics*, 19, 2-10.

Clark-Wilson, A. (2010). Emergent pedagogies and the changing role of the teacher in the TI-Nspire Navigator-networked mathematics classroom. *ZDM,42(7)*, 747-761.

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom, *Educational Studies in Mathematics*, *75(2)*, 213-234.

Olsher, S., Yerushalmy, M., & Chazan, D. (2016). How might the use of technology in formative assessment support changes in mathematics teaching? *For the Learning of Mathematics, 36(3),* 11-18.

Panero, M., & Aldon, G. (2015). How can technology support effectively formative assessment practices? A preliminary study. In N. Amado & S. Carreira (Eds.) proceedings of the 12th International Conference on Technology in Mathematics Teaching (ICTMT 12). 293-302.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students’ command process through instrumental orchestrations. *International Journal of Computers for mathematical learning, 9(3)*, 281-307.

Yerushalmy, M. & Elikan, S. (2010). Exploring reform ideas of teaching Algebra: Analysis of videotaped episodes and of conversations about them In R. Leikin & R. Zazkis (Eds.), *Learning through Teaching: Developing mathematics teachers' knowledge and expertise in practice*. (pp. 191-207). Springer Netherlands.