

# COMPETENCIES AND DIGITAL TECHNOLOGIES – REFLECTIONS ON A COMPLEX RELATIONSHIP

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Advantages and disadvantages of the use of digital technologies (DT) and especially of computer algebra systems (CAS) in mathematics lessons are worldwide discussed controversially. Many empirical studies show the benefit of the use of DT in classrooms. However, despite of inspiring results, classroom suggestions, lesson plans and research reports, the use of DT – and especially CAS – has not succeeded, as many had expected during the last decades. The thesis of this article is that we – the researchers and teachers who are interested in the use of DT since many years or decades – have not been able to convince teachers, lecturers at university and parents of the benefit of DT in the classrooms in a sufficient way. In the following, the working with DT will be related to understanding and classroom activities. The basis of the argumentation is a competence model, which classifies – for a special content – the relation between levels of understanding (of the concept), representations of DT and different kinds of classroom activities.

*Keywords: digital technologies, tool-competencies, representations, classroom activities, calculus.*

## CONCERNING THE USE OF DIGITAL TECHNOLOGIES (DT) IN MATHEMATICS CLASSROOMS

There are many theoretical considerations, empirical investigations and suggestions for the classroom concerning the use of DT in mathematical learning and teaching (e. g. Guin et.al., Zbiek 2007, Drijvers & Weigand 2010, Weigand 2013). In recent times, some empirical studies started integrating DT and especially computer algebra systems (CAS) into regular classroom teaching and covering longer periods of investigation. E. g. the *e-CoLab*<sup>1</sup> (Aldon et al. 2008), *RITEMATHS*<sup>2</sup>, *CALIMERO*<sup>3</sup> (Ingelmann and Bruder 2007), *M<sup>3</sup>-Project*<sup>4</sup> (Weigand 2008, Weigand and Bichler 2010b, Weigand & Bichler 2010c). The main results of these projects and investigations can roughly be summarized as follows: DT (and especially CAS)

- allow a greater variety of strategies in the frame of problem solving processes;
- are a catalyst for individual, partner and group work;
- do not lead to a deficit in paper-and-pencil abilities and mental abilities (if these abilities are regularly supported in the teaching lessons)

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<sup>1</sup> e-CoLab = Expérimentation Collaborative de Laboratoires mathématiques. See:

<http://educmath.inrp.fr/Educmath/dossier-parutions/experimentation-collaborative-de-laboratoires-mathematiques>. Accessed 23 February 2017

<sup>2</sup> RITEMATHS = The project is about the use of real problems (R) and information technology (IT) to enhance (E) students' commitment to, and achievement in, mathematics (MATHS). <http://extranet.edfac.unimelb.edu.au/DSME/RITEMATHS>. Accessed 23 February 2017

<sup>3</sup> CALIMERO = Computer Algebra in Mathematics Lessons: Discovering, Calculating, Organizing (translated title).

<sup>4</sup> M<sup>3</sup> = Model Project New Media in Mathematics Education

- allow more realistic modelling problems in the classroom (but also raise the cognitive level of the understanding of these problems);
- do not automatically lead to changed or modified test and examination problems (compared to paper-and-pencil tests);
- demand and foster advanced argumentation strategies (e.g. if equations are solved by pressing only one button).

Overall, Drijvers et.al. (2016) concluded from a meta-study-survey of quantitative studies, that there are “significant and positive effects, but with small average effect sizes” (p. 6), if for the benefits of integrating DT in mathematics education is asked. Moreover, there is also a broad consensus, that gainful changes in classroom teaching and learning need didactic and methodic considerations and a thorough thinking about the goals of teaching and learning.

## VISIONS AND DISILLUSIONS

The first ICMI study in 1986 “The Influence of Computers and Informatics on Mathematics and its Teaching” (Churchhouse) was affected by a great enthusiasm concerning the perspectives of mathematics education in view of the availability of new technologies. However, in the ICMI Study 17 “Mathematics Education and Technology – Rethinking the terrain” (Hoyle & Lagrange 2010) and the OECD Study (2015), “Students, computers and learning. Making the connection”, disappointment is quite often expressed about the fact that – despite the countless ideas, classroom suggestions, lesson plans and research reports – the use of DT has not succeeded, as many had expected at the beginning of the 1990s.

Worldwide, the current situation concerning the use of DT is very versatile. There are countries (like Norway or Denmark) that are intensively using laptops, tablets (with the programs Geogebra or Maple) or symbolic calculators (like the TI-Nspire or the Casio Classpad). These countries even allow using these tools in examinations. There are other countries (like the UK or France) that allow “only” symbolic calculators in examinations, there are countries – especially in Asia – which are very sceptical about the use in examinations, and there are countries (like Germany) where there are a different situations about the use of DT – depending on the state.

Reflecting the developments of the use of DT in the last decade, the results concerning the possibilities of supporting students’ learning processes have been started to rethink and then especially raised the question: What is the benefit of using DT in the classroom? (Weigand 2017). More specifically important questions are:

1. In relation to which mathematical contexts does the use of DT make sense and which (mathematical) competencies are supported and developed?
2. Which mathematical and tool competencies are necessary, or at least helpful, when working with DT for specific mathematics content?
3. How can the DT-use be described in a more detailed form?

In the following it will be tried to give answers to these questions, by constructing a model that shows the relation between working with DT, different levels of understanding and specific activities in the classroom. The result is a three-dimensional competence model for DT-use.

## COMPETENCE MODELS

### Theoretical Foundations

The concepts of *competence* and *competence (level) models* have aroused interest in mathematics education in the past years. Starting with the NCTM Standards (1989) and especially the PISA

studies, *competence* and *competencies* are expressions, often used in the context of standards and substituted the “old expression” *goals* which envisaged knowledge and abilities in mathematics education. “Mathematical competence means the ability to understand, judge, do, and use mathematics in a variety of intra- and extra-mathematical contexts and situations in which mathematics plays or could play a role ...” (Niss 2004, p. 120). In the PISA studies, competencies are on the one hand related to the *content*, e.g. numbers, space and shape, change, etc., and on the other hand – in a more general way – related to *processes* like problem-solving, modelling and the use of mathematical language. In order to evaluate or operationalize the competencies through the construction of items and tests, it is helpful to organize these competencies in levels, categories or classes. In the PISA studies, each of the possible pairs (content, process) can be divided into three different levels or competence classes (OECD 1999, p. 43): Class 1: reproduction, definitions, and computations; Class 2: connections and integration of problem solving; Class 3: mathematical thinking, generalisation and insight. This leads to a three-dimensional competence-model with the *dimensions* content, basic or process competencies and cognitive activation.<sup>5</sup>

### Competence model for symbolic calculators while working with functions

In Weigand and Bichler (2010a) a competence model for the use of symbolic calculators with CAS in mathematics lessons in the frame of working with functions was developed. It can also be extended to the use of DT overall. Different levels of *understanding the function concept* have been seen in relation with the *representations* and – as a third dimension – with *cognitive activation*. These levels of understanding are not strictly hierarchical, because they are intertwined during the developing process, e. g. conceptual aspects have to be seen in relation to intuitive and relational aspects.

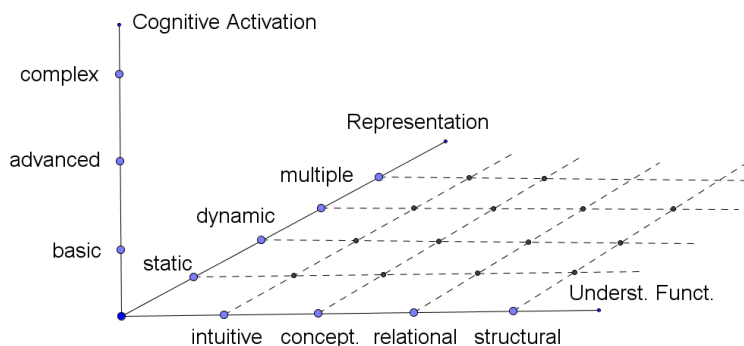


Fig. 1. Competence model for DT while working with functions

The ability or the competence to adequately use the tool requires technical knowledge about the handling of the tool. Moreover, it requires the knowledge of when to use which features and representations and for which problems it might be helpful. Three levels are distinguished, which might also be categorized by using DT as a (simple) function plotter, as a tool for creating dynamic animations and as a multi-representational tool.

### Competence model for DT-use in the classroom

The model in Fig. 1 is gainful if tasks and problems have to be classified, e. g. for tests and examinations. It does not adequately fit if activities in the classroom should be integrated and evaluated. In the following model the second dimension “Representation” was changed due to the well-established theory of representation, which emphasizes the reasoning with multiple and dynamic representations (Bauer 2013 or Ainsworth 1999). Moreover, understanding and working

<sup>5</sup> In PISA, these dimensions are called “Overarching ideas” (content), “Competencies” (process) and “Competence Clusters” (cognitive activation).

with representations are seen in relation to classroom activities. A third dimension with the following activities will be introduced:

- Calculate: DT as a tool for (numeric and symbolic) calculations, and especially CAS are tools, which allow calculation on a symbolic level in notations close to the mathematical language. Example: Seeing parameter dependent functions as functions of several variables allows an efficient working in problem solving processes.
- Consult: DT – especially CAS – as a consultant in the sense of using a formulary. Example:  $(a + b)^3 = a^3 + \dots$ .
- Control: DT as a controller of hand-written solutions, suggestions and ideas on a graphical, numeric or symbolic level.
- Explain: DT are catalysts for the communication between the user (student, learner) and someone who has to interpret or understand the DT-solutions (e.g. a teacher). DT are sources for explanations and argumentations.
- Discover: DT as a tool for evaluating and testing suggestions and strategies in a problem solving process.

This classification may be seen as a hierarchy while moving from a procedural knowledge (Calculate) to a conceptual knowledge (communicate, discover). Of course, also the activity “levels” are intertwined and do not represent a strict hierarchy. This new third dimension is more on the teaching side while the dimension “Cognitive activity” (Fig. 1) is more on the learning side.

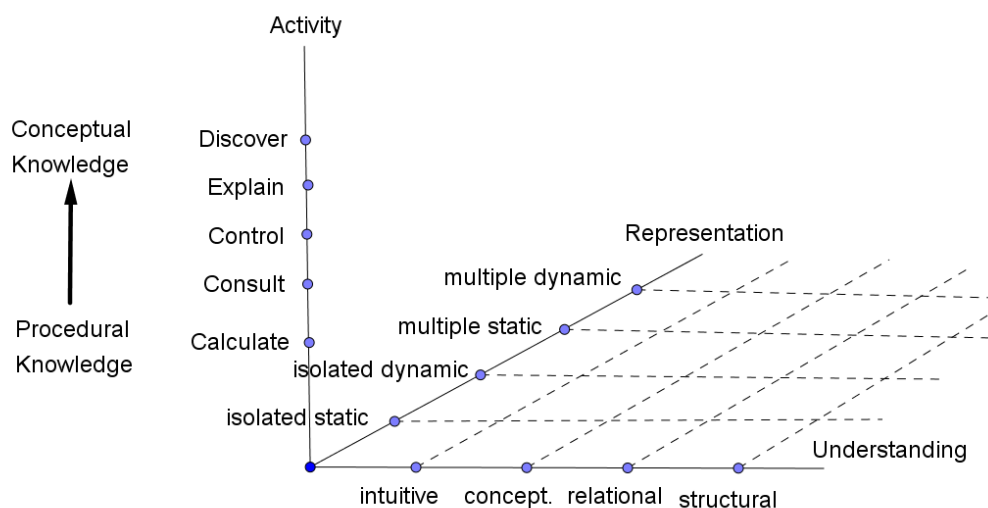


Fig. 2. An extended competence model integrating (classroom) activities

This competence model has three categories and gives us  $4 \times 4 \times 5 = 80$  cells. If each cell is again subdivided into three levels of cognitive activation, this makes a total of 240 cells and this is only for a special concept. This already shows that it is very difficult or even impossible to create special examples for each of these cells. This competence model is more for pointing out the directions and goals concerning understanding, the kinds of used representations, and the kinds of activities DT might be used for adequately develop a special concept.

What is meant by *tool competence*? A *tool* as “something you use to do something” (Monaghan et.al. 2016, p. 5) is the quite general definition. *Mathematical tools* allow us to create, to operate with and to change mathematical objects. DT and CAS are *digital tools*. The word *tool* is used

instead of *instrument* because the facilities of DT in relation to mathematics aspects in the classroom are in the foreground, and the development of the user-tool-relationship in the frame of an instrumental orchestration, which is the heart of the instrumental genesis (see Artigue 2002, Drijvers et.al. 2010), is left to the user or learner. *Tool-competence* is the ability to refer the competence-model (Fig. 2) to a special concept. Tool-competence describes the development of the understanding of a concept in relation to the tool-representation in the frame of classroom-activities.

While empirical competence models – like the PISA model – help to answer the question *whether* students or learners do benefit from special learning or teaching interventions, the model (Fig. 2) is more process-oriented and should give reasons *why* and *how* this might be the case.

The problem of this model is the specification of the 80 cells with prototypical examples. It is not possible to construct a one-to-one-relationship between a cell and a special example. Problems, situations or examples can mainly or even always be seen under different aspects and never relate only to one level of understanding or to one level of activity. The challenge with regard to an empirical justification of this model is the construction of prototypes of examples which emphasize the triad of one cell of this model.

## EXAMPLES

In the following the concentration is on CAS, the fields these tools are mostly used and DT promise the biggest changes compared to traditional courses: algebra and calculus. In this article the restriction on a few spotlights of examples is necessary.

### Functions

Working with functions on a structural level means to see functions as objects, to use symbols like  $f$  and  $g$  on the symbolic level, to e.g. add  $(f + g)$  and multiply  $(f \cdot g)$  them, and to represent them in different representations. It is expected that learners e. g.

- can work with functions on different levels of understanding;
- can work with functions as objects on a symbolic and a graphical level; they especially interpret changes of variables of a function as geometrical transformations;
- understand the definition of functions of several variables and they can– adequately to the situation–interpret them as functions of one variable with parameters;
- can use functions of several variables to solve mathematical and modelling problems.

### Sequences

Working with *recursively-defined sequences* with  $a_{k+1} = f(a_k)$ ,  $k \in \mathbb{N}$ , a first element  $a_1 \in \mathbb{R}$  and a function  $f: \mathbb{R} \rightarrow \mathbb{R}$ , CAS allow to calculate the *sequence of iteration*

$$a_1, a_2 = f(a_1), a_3 = f(a_2), \dots,$$

and to represent it numerically and graphically as  $k$ - $a_k$ -diagrams or “cobweb-diagrams”. This can be done on a conceptual level of understanding and multiple dynamic representations can be used.

*Difference sequences*  $(\Delta a_k)_{\mathbb{N}}$  with  $\Delta a_k := a_{k+1} - a_k$  and a given sequence  $(a_k)_{\mathbb{N}}$  are well suitable for a discrete introduction of the difference quotient (see Weigand 2015).

Overall, the meaning of CAS concerning the content *sequences* can be summarized like the following: CAS

- are tools with notations (or a language) quite close to mathematical notations (or the mathematical language);

- allow symbolic calculations and show related numeric and graphic representations;
- allow object-related working with sequences and discrete functions;
- have to be seen or evaluated in relation to other—especially graphical—representations.

## Equations

A CAS is a formulary that offers in particular solution formulas for linear and quadratic equations and for systems of linear equations. In relation with a graphic representation, questions concerning the *number* of zeros of a quadratic function can (at first) be answered through experimental exploration. A CAS can be used to calculate the zeros of a function by only pressing one button, but moreover, it serves as visualization. Furthermore, the relation of function and equation is fundamental for the mutual representation in the CAS and the graphic window.

If the CAS is used tool for solving systems of equations with parameters, learners work on a structural level of understanding with multiple dynamic representations.

Example 1: Systems with quadratic equations can be calculated (on a symbolic and graphic level)

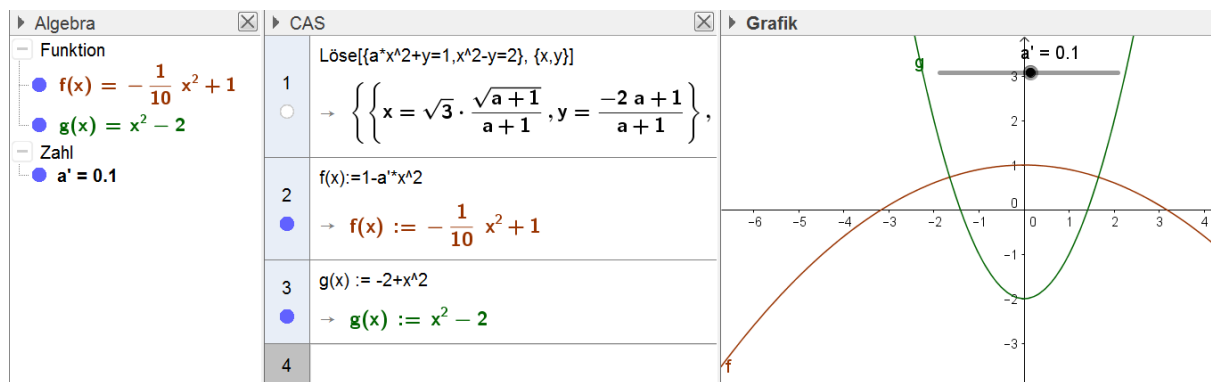


Fig. 3. Solving a system of quadratic equations with one parameter

The CAS provides calculations and solutions on the symbolic level and these have then to be interpreted, especially in relation to the graphical level.

Example 2: The symbolic solutions of more complex equations like  $x^3 - x + 1 = 0$ ,  $1 + \sin(x) = 2^x$  or  $x^7 - 4x^5 + 4x^3 = 0$  depend on the equation. But an efficient use of a CAS is only possible if it is based on mathematical knowledge concerning the solution of equations, the characteristics of the underlying functions of the equations and the possibilities of the solution varieties. For calculations the CAS is used mainly within the static isolated symbolic representation, adding graphic representations for interpreting or explaining symbolic results. The advantage of using CAS is the notation of solutions on a symbolic level, especially while working with equations with parameters. The communication with the tool is possible in a language close to the traditional mathematical language. The CAS is a consultant in the sense of a formulary for symbolic solutions especially for polynomial equations of order 2 or 3.

## CONCLUSIONS

The developed competence model is a theoretical or normative model. It applies the understanding of a concept to the working in the classroom and the use of a tool (with different representations). It is a model for the evaluation of the process and development of understanding of a special concept in a tool-supported classroom.

Concerning the empirical justification of the theoretical model and with the aim of constructing an *empirical* competence model, some questions have to be answered.

1. *Task development*: Tasks and appropriate learning environments concerning a special concept for the levels of understanding and kinds of activities have to be developed, which promise a benefit while working with DT compared to traditional paper and pencil working. Will the learners be able to work adequately (in the theoretically expected way) with different kinds of representations and do they develop a normative expected understanding on different levels?
2. *Micro-connectivity of the tool-use*: Working with a CAS on the symbolic level has to be seen in relation with other representations, especially with numerical and graphical representations (the aspect of multiple representations). These additional representations allow interpretations of symbolic results and expressions. Which representations are adequate on which levels of understanding and which kinds of activities?
3. *Dynamic aspects*. The dynamics of representations have to be seen in relation to the dynamic aspects of variables, and in consequence to the dynamics of the concepts of function, equation, derivative, ... How is the transition from static to dynamic representations and how is it related to levels of understanding and kinds of activities?
4. *Diagnostic instrument*: The competence model might also be used for diagnostic reasons to evaluate the “tool-competencies” of special learners (or one special learner). Diagnostics are the first step while improving students’ understanding; the second step is to establish consequences to *improve* these. How can a learner be supported the best way to attain tool-competencies?
5. From a qualitative to a quantitative model: The PISA studies use a model with a numerical competence scale, which is based on the relative frequency with which students are able to solve a problem. The problem that has been solved successfully is taken as a measure of the difficulty of the exercise. The scale is standardized on a mean value of 500 with a standard deviation of 100 (OECD 2003). This might also be an aim in the context of this competence model. How can the competence model (Fig. 2) be extended or transformed to a quantitative model?

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<sup>7</sup> [http://www.pisa.oecd.org/document/29/0,3343,en\\_32252351\\_32236173\\_33694301\\_1\\_1\\_1\\_1,00.html](http://www.pisa.oecd.org/document/29/0,3343,en_32252351_32236173_33694301_1_1_1_1,00.html)