Dynamic Technology for Simulating a Scientific Inquiry for Learning - Teaching Pre-Calculus concept

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In the proposed workshop, we are interested to share with mathematics educators a possible use for dynamic technology to simulate a scientific inquiry for teaching-learning pre-calculus topics. Inspired by the logic of inquiry approach and the variation theory, we designed a set of activities and methodology that aim to bridge the gap between the formal world of mathematics and the real-life situations. Participants of the workshop will experience the activities and will engage in the methodology for conducting classroom practices. Results from our ongoing research would be shared with the participants and insights could be discussed that would emerge from the experience.

Rational

Calculus is considered to be one of the most important topics in mathematics, and it is included in several curricula worldwide. Calculus - the study of how things change - is inherent in many other topics and in many real-life situations. It is difficult to imagine that there could be any other dynamic phenomena which is unable to be modelled by means of calculus. Nurturing of calculus thinking can result in the productive integration of citizens into modern society. Exposing students to the logical structure of calculus concepts may help develop the logical thinking of the students. This type of thinking is essential in dealing with the challenges that citizens face in the 21st century. Furthermore, encouraging students to model mathematically real-life situations may help them integrate within society as citizens who are able to make intellectual decisions. Our research is aimed at understanding the ways of teaching-learning of pre-calculus concepts in a technologically rich environment that simulate a scientific inquiry of real-life situations.

Often, as seen in Italy as well as in Israel, calculus is taught in the upper secondary schools in a formal way as a set of rules and strategies for investigating functions and computing areas bound between functions. This kind of teaching-learning essentially concentrates on the formal world of mathematics which poses a barrier for the sense of mathematical statements (Arzarello, 2016a, b). Several scholars have criticized this kind of teaching and claimed that it is not a guarantee for boosting the understanding of calculus concepts, and even found it to be a barrier for understanding calculus when it is learned at a university level (e.g. Thompson et al; Swidan & Yerushalmy, 2016; Broussard, 2011). Also the Italian official curriculum (www.indire.it/lucabas/lkmw_file/licei2010/indicazioni_nuovo_impaginato/_decreto_indicazioni_nazionali.pdf) points out the necessity of presenting examples where mathematical models of different phenomena are emphasized.

To overcome the barrier that existed between the formal world and the real-life situation, Arzarello (2016a) has suggested a ‘virtuous cycle’ model. The model consists of four formal and informal intertwined aspects: (1) Aspects of the real situation represented in the formal system. (2) Treatment within a formal system / Conversions between systems. (3) Interpretation of the results of the formal system in the real situation. And (4) Interpretation/theorization of the real situation through the theoretical lens. Since the formal and informal aspects are deeply intertwined in the mathematical reasoning, Arzarello (*ibid.)* argued that a major teaching goal should be to operationalize this virtuous cycle in classroom practices.

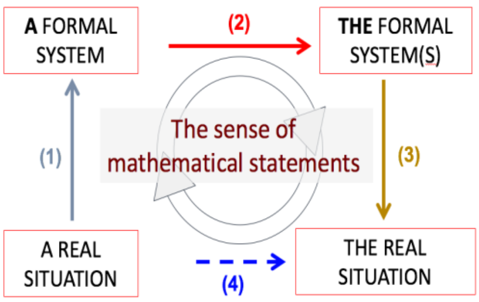

Figure 1. Virtuous Cycle (Adapted from Arzarello (2016a))

However, the main question remains as to how to apply the virtuous cycle in classroom practices, that ensures a deep understanding of the mathematical concepts. This question should guide the discussion of the workshop. Inspired by the logic of inquiry approach (Hintikka, 1998), which generally viewed scientific inquiry and knowledge construction as a question-answer process, and the variation theory (Marton and Tsui, 2004) which defines learning as a change in the way something is seen, experienced or understood. We have designed a set of activities and suggest a methodology (Method of Variation Inquiry) which may facilitate the conceptual learning of pre-calculus concepts and to engage students in a scientific inquiry.

Hooke's Law Activity

Task 1

Your task is to watch the Hooke's Law clip (Fig. 2) and to answer the following questions:

What attracted your attention when you watched the clip? Write as many observations as you can?

After watching to the Hooke's Law clip, write as many hypothesis as possible about the content of the clip, which you may wish to discuss later.

Task 2

Your task is to explore how the change of the mass affects the extension of the spring.

A. Can you make a conjecture about how the change of the mass should affect the extension of the spring?
B. Open the Hooke's Law **1 applet** (Fig. 3). Change the mass to verify or refute the conjectures you raised in (A). Did your conjectures change? If yes, why? If not, justify and prove your conjectures.
C. How do the differences between the y values of the points on the graph change when varying the the mass. Why? Is your conjecture always true? Can you prove it?
D. Can you find an equation that represents the sketch of the spring? Why or why not? Justify your answer.

Task 3

Your task is to explore how the change of the spring elasticity affects its extension.

A. Hypothesize how the elastic of the spring affects its extension. Why?
B. Open the Hooke's Law **2 applet** (Fig. 4), vary the elasticity of the spring, and change the mass. Refute or verify the hypotheses you raised in (A). Justify and prove your hypotheses.
C. Why does the function graphs change as the elasticity of the spring varies? Discuss with your classmates which aspects changed and which ones remained invariant.
D. Follow your interaction with the applet. Write new hypotheses based on your experience with the applet. Which hypotheses can and can't you justify? Why?

Task 4

Challenge your classmates by asking them questions. You win the game if you ask a question about the Hooke's Law, which your classmate cannot answer. Use either applet to ask your questions.

Figure 1. Hook's Law activity that was given to the students

Figure 2. Video clip that the students watch as the starting point for the inquiry

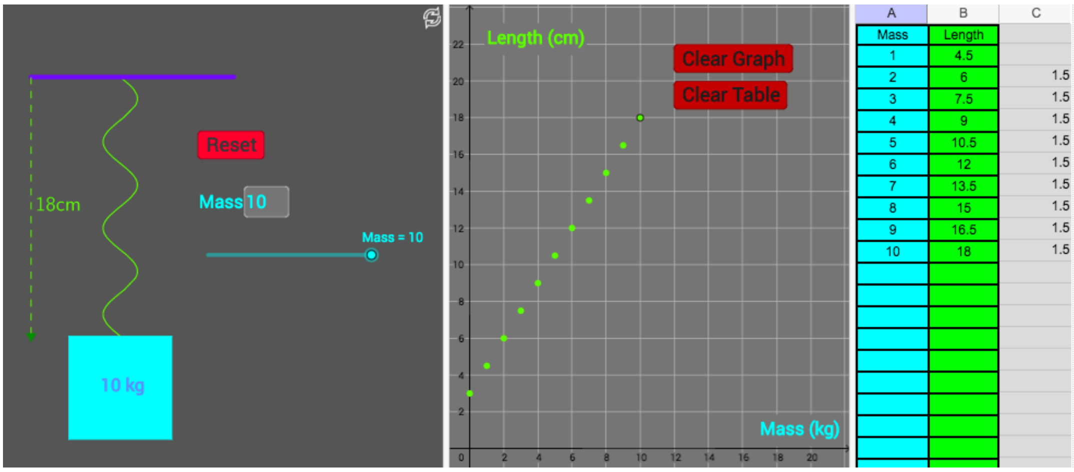

Figure 3. Interface of the Hooke's Law 1 applet

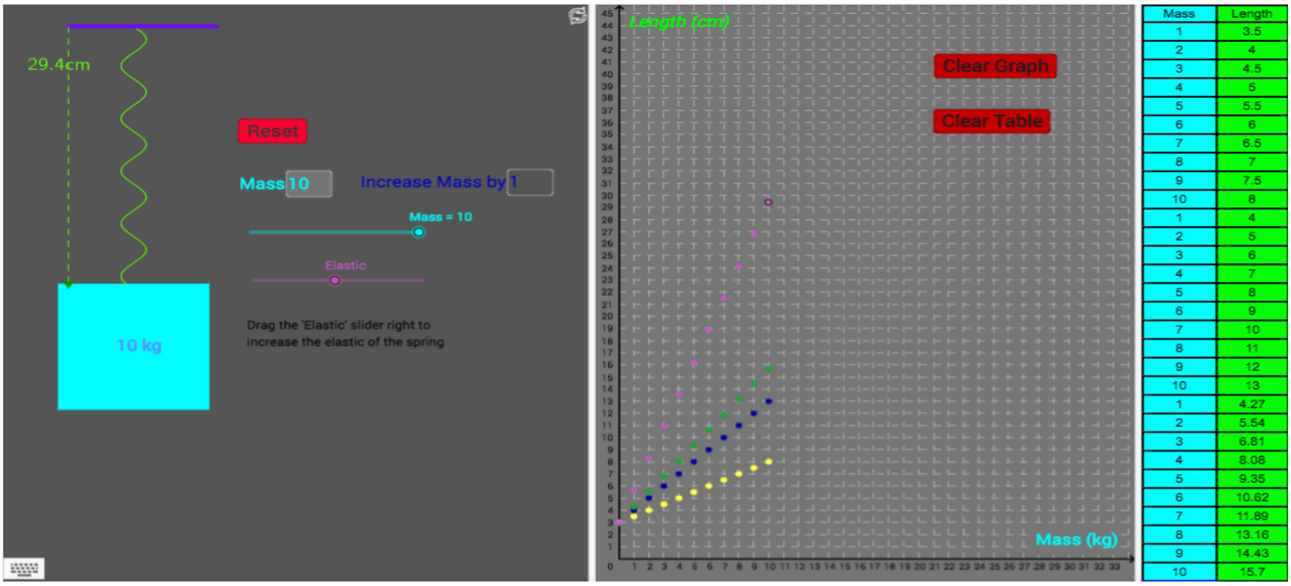

Figure 4. Interface of the Hooke's Law 2 applet

The Method of Variation Inquiry (MVI) is based on the idea of creating inquiry modes for students by changing and challenging the observations they make during the learning process. The MVI model consists of six levels:

- Level 0. Choosing a starting point.
- Level 1. Listing the observations.
- Level 2. Collective discussion (asking questions, reacting to observations). The conclusion should be If (Oi) then (Dj). The explanation why the sentence is or is not correct is shared.
- Level 3. What if the statement is not correct? (~Oi)
- Level 4. Collective discussion: the conclusion should be If (~Oi) then (Dj)*. The explanation why the sentence is or is not correct is shared.
- Level 5. Meta-reflection:
  - If (Oi) then (Dj)?→ (Rk)
  - If (~ Oi) then (Dj)* →(Rk)*

Questions and Goals

The workshop will address the question of how the designed activities and the suggested methodology could be applied in classrooms across different cultures. In addition, epistemological and cognitive opportunities as well as obstacles that are raised during the application of the proposed activities and methodology in classrooms could be discussed. We will use examples from our ongoing research to provoke discussions of these questions. These examples are derived from our analysis of a beta experiment conducted in a scientific oriented school in Italy, in which 10^{th} grade students are working in small groups as well as a teacher who conducts a collective discussion using the suggested methodology. In our beta experiment, we have noticed learning opportunities that encouraged a deep understanding of pre-calculus ideas. We hope participants in the workshop will notice additional opportunities (obstacles) which would allow them to engage in discussions of the implications of such results for improving the teaching-learning pre-calculus concepts in classrooms.

Planned Activities

a.	Introducing the 'Method of Variation Inquiry': theory and practice.	15 min
b.	Experiencing the methodology in small groups through solving the designed activities.	20 min
c.	Discussing the opportunities and obstacles of the methodology and the activities.	15 min
d.	Introducing the finer logic of inquiry model: theory and practice	20 min
e.	Analyzing transcripts using the finer model of logic of inquiry	20 min
f.	Collective discussing of how the methodology and the activities could be applied in classrooms	20 min

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