CENTRAL AND PARALLEL PROJECTIONS OF REGULAR SURFACES: GEOMETRIC CONSTRUCTIONS USING 3D MODELING SOFTWARE

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The contribution addresses the constructions of central and parallel projections of regular surfaces which can be regarded as a subarea of descriptive geometry. My aim is to increase the interest of students in classical and descriptive geometry primarily through 3D computer modeling. I have been seeking to establish a stronger connection between descriptive geometry and its practical applications and to extend descriptive geometry with knowledge of computer graphics and computer geometry. In order to provide insight into more complex geometric problems and to increase the interest in geometry, I have integrated 3D computer modeling in my lectures and seminars. Geometry is a necessary component of many engineering processes such as the development of innovative graphics software or the design of complex industrial and architectural structures. My aim is to show that the principles and knowledge of classical and descriptive geometry are the stepping stones for solving tasks in practice.

Keywords: central and parallel projections; regular surfaces; descriptive geometry; 3D computer modeling; computer geometry

MOTIVATION

I have been teaching classical geometry, descriptive geometry and computational geometry at the Faculty of Mathematics and Physics (Charles University) in the Czech Republic for several years. I also supervise bachelor and master theses on various geometric topics. The motivation for studying geometry can be found in many branches (building practice, engineering and construction practice, architectural and industrial design, production industries …) and geometry represents one of the highly demanding fields of mathematical science which require logical thinking and which also strongly stimulates spatial imagination. The study of geometry represents an ongoing challenge in terms of research and practice.

In this contribution I will focus on modern teaching methods of descriptive geometry and I will put these methods in contrast to traditional hand sketching and drawing activities. Computer-aided education of geometry will be demonstrated on examples of central and parallel projections of regular surfaces.

Descriptive geometry (Paré et al., 1996; Pottmann et al., 2007; Robertson, 1966) represents a subarea of classical geometry and deals with the representation of three-dimensional objects in two dimensions. The typical task in descriptive geometry is to represent three-dimensional objects on a two-dimensional planar surface or to reconstruct three-dimensional objects from the two-dimensional image i.e. result of the projection. Descriptive geometry deals with those representations which are one-to-one correspondent. From a historical point of view, the development of descriptive geometry reached its greatest height in the last century. Nevertheless, even despite today's innovative approaches and continuous development of modern computer technology and equipment, descriptive geometry has not lost its importance. The role of descriptive geometry in practice is irreplaceable in such branches in which correct visualization is crucial. To be

able to project some three-dimensional object and get the two-dimensional result the construction methods require good knowledge of the fundamental geometry, the properties of geometrical objects in the plane and in the space, and their relations. This means that the study of descriptive geometry represents the significant stepping stones for solving geometric tasks in real practice.

In my research, I investigate innovative methods of explaining complex concepts in teaching of geometry (specifically descriptive, classical, computational geometry) at Czech colleges and their impacts on students' successes. The innovation in explanation and didactic methods include 3D computer modeling and interactive software visualization. My aim is to stimulate the interest of students in geometry, to increase their motivation, to improve their understanding of geometry, to improve the methods of teaching geometry currently in use, to help students achieve better results in examinations and to promote the practical use of geometry. I seek to make traditional topics from classical and descriptive geometry more attractive to students by updating the current methods of teaching geometry. The new teaching methods are aimed at strengthening the connection between classical geometry and the practical applications thereof on the one hand and extending classical and descriptive geometry into computer graphics and computational geometry on the other. The connection between classical geometry and 3D computer modeling is intuitively understood by students.

In this contribution I will describe possible activities, examples of outputs from 3D modeling software and the combination of descriptive and computational geometry. I will explain what computer-aided education of geometry means in my lessons. I have been using described activities in university classroom practice for several years. The main research question within this article is to discuss either use 3D computer modeling software for creation of 3D models and animations or hand sketching and drawing activities within specific task in education of descriptive geometry or how to combine them. The possibilities of 3D computer modeling in education of geometry will be shown and discussed.

I also find very important to clarify what is my aim for the future. I plan to deal with the following explorations and gather the results regarding computer-aided education in university classroom:

- the evaluation of success rate of university students attending geometric courses in the last few years (before and after when the innovative methods were implemented),
- questionnaire survey which was conducted among university students attending the courses and lectures on geometric topics where computer aided education was realized,
- my survey revealed that the modern type of computer-aided education was adopted very positively among students and according to higher students' interest in geometric topics within the research projects and qualification theses they seem to be more motivated,
- using computers in education of geometry is an efficient aid because geometrical and mathematical software (*GeoGebra*, *Rhinoceros*, *Mathematica*, *Maple*,…) allow us to deal with more complex task even in classroom practice,
- proper functions and tools in geometrical software develop creativity and imagination of students; on the other hand, the ability to use geometrical or mathematical software is not equal to the knowledge of geometry.

The evaluation of these surveys is a very complex and long-lasting task but first results show that our efforts to improve geometry education are successful. Some results of my didactic survey have been already published, (Surynková, 2013, 2015).

In this article we will show the examples of using modeling and graphics software in teaching geometry. Computer-aided education of descriptive geometry is demonstrated on typical tasks from descriptive geometry - the constructions of the two-dimensional results of central and parallel projections.

The rest of the paper is organized as follows. In Section 3D modeling software versus traditional teaching methods of descriptive geometry we introduce novel methods of using modern software in a classroom practice and remind the importance of traditional approaches including hand sketching and drawing. In Section Parallel and central projections of regular surfaces we show examples of geometric tasks regarding these topics including students' work. Discussion, summary, and future work are given lastly.

3D MODELING SOFTWARE VERSUS TRADITIONAL TEACHING METHODS OF DESCRIPTIVE GEOMETRY

Geometry in general ranks among the demanding subjects in secondary schools and colleges. One possibility how to stimulate the interest of students in geometry is to show them that the challenges of this Information Age can be addressed by means of geometry. The practical applications of geometry include computer-aided architectural and industrial design. Geometry is essential to the manufacturing, engineering and construction industries; the digitization of real objects using 3D scanning; digital surface reconstruction from point clouds; the replication of the shapes of real-world objects using 3D printing; computer graphics and many more, (Eilam, 2005; Foley et al., 1995; Hoschek and Lasser, 1993; Lipson et al., 2013; Sarkar, 2015). All these applications can be characterized by combinations of geometric principles. The extension of descriptive geometry, and geometry in general, into 3D computer modeling is a very promising approach how to increase student's motivation and to improve the methods of teaching geometry currently in use.

There exist a wide range of professional graphics software and environments which provide the required user input tools, and speed up production and are commonly used in the process of designing, design documentation and construction for modeling and drawing (Farin et al., 2002). We can use similar software in teaching of descriptive geometry.

I have integrated 3D computer modeling in my descriptive geometry lessons and seminars and I work mainly with the *Rhinoceros* (*NURBS Modeling for Windows*) software which is a commercial NURBS-based 3D modeling tool, (McNeel, 1999), commonly used in the process of designing, design documentation and construction. It is not necessary to work only with *Rhinoceros* or with expensive CAD applications. As there exist the number of inexpensive or free software applications for geometry and mathematics, students and teachers can use them. One of the most widespread free geometrical tools is mathematics and geometry dynamic software *GeoGebra*. The great advantage of *GeoGebra* is the possibility to change dynamically the parameters of the designed geometrical objects.

I have been teaching descriptive geometry and related subjects for more than ten years and my personal experiences show that teaching and studying geometry must be accompanied by traditional methods i.e. hand sketching and drawing on the blackboard and on the sheets of paper. Drawing and sketching helps us to develop our precision skills and patience and we rely on these tools when developing of our initial ideas and finding solutions to geometrical problems. In my lectures and seminars I combine the both approaches to the teaching of descriptive geometry - the traditional geometry teaching methods and procedures (sketching and drawing activities) and modern computer-aided education using digital modeling tools.

PARALLEL AND CENTRAL PROJECTIONS OF REGULAR SURFACES

Let us show the examples of concrete topics in descriptive geometry where 3D computer modeling can be used. I use 3D computer modeling to create 3D models of geometric objects and situations in the three-dimensional space, I demonstrate geometrical constructions of regular surfaces and I also use digital tools for creation of central and parallel projection of surfaces. I use these outputs during my lessons as illustrations of geometrical properties of studied objects which can help my students understand geometrical problems in intuitive and natural way. Students can discover geometrical principles and properties of objects more easily.

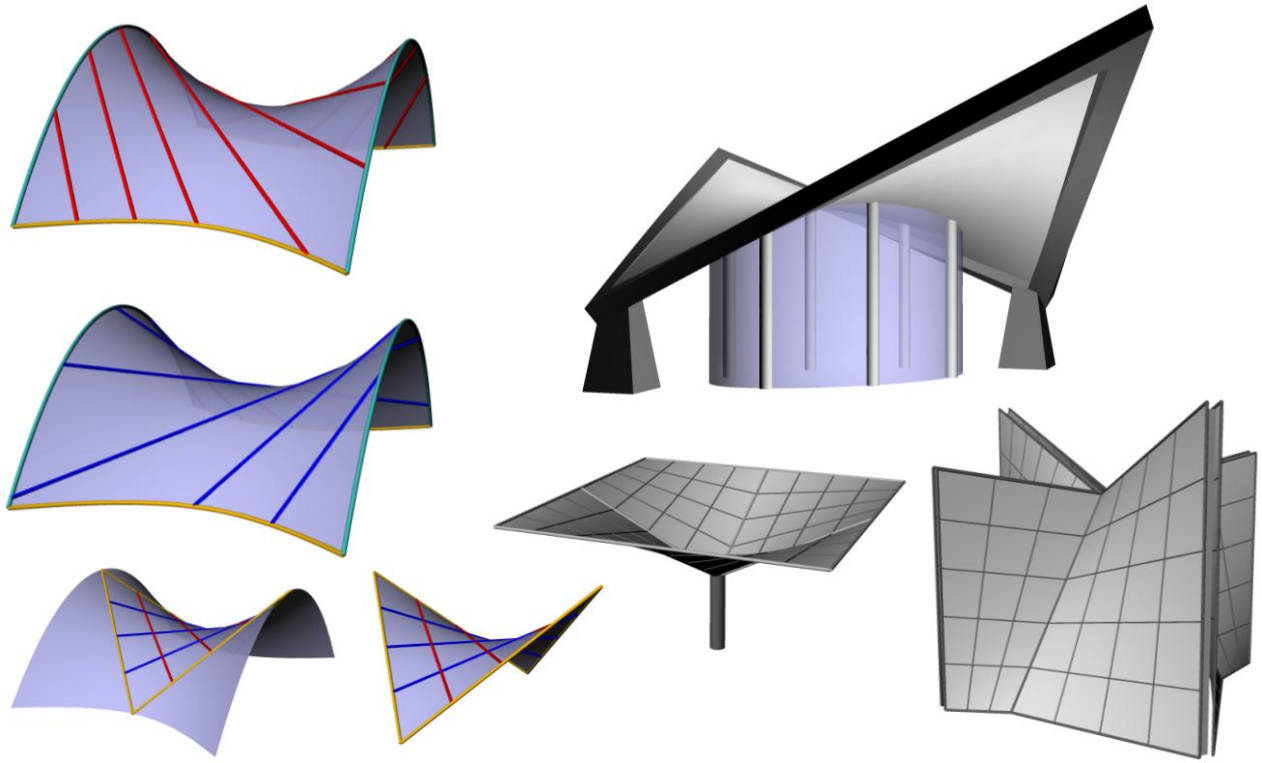

Figure 1: Ruled surfaces - determination and practical applications.

When I am teaching the topic of regular surfaces, firstly, a theoretical explication regarding the determination of surfaces in the three-dimensional space is provided and here the illustrations from the 3D computer modeling software can be used with potentially great success. The virtual model of the spatial situation and 3D virtual models of surfaces make a significant contribution to the development of spatial imagination. Besides the determination and the properties of regular surfaces I also present the practical usage of studied surfaces to students. Figure 1 shows an example of ruled surfaces. Firstly, the illustration of the determination of a surface is provided; secondly, the practical usage of this type of surfaces is shown. All these pictures are created as three-dimensional objects in the *Rhinoceros* software. So when I am teaching, I can show these 3D models to my students directly in 3D modeling software, I can change the view of a designed object and we can observe the spatial objects from different positions.

As has been already pointed out, my aim is to integrate the knowledge of computer graphics and computer geometry into my descriptive geometry lessons. Figure 2 shows an example of the determination of a ruled surface using *GeoGebra*. In *GeoGebra* we can use the animation for the demonstration of sequential construction of a surface. In this case it is necessary to know the

mathematical description of a studied surface. So creating all these illustrations does not mean only "drawing nice pictures"; we have to know how to describe a surface in software environment. We can for example use a parameterization or synthetic construction of studied surfaces.

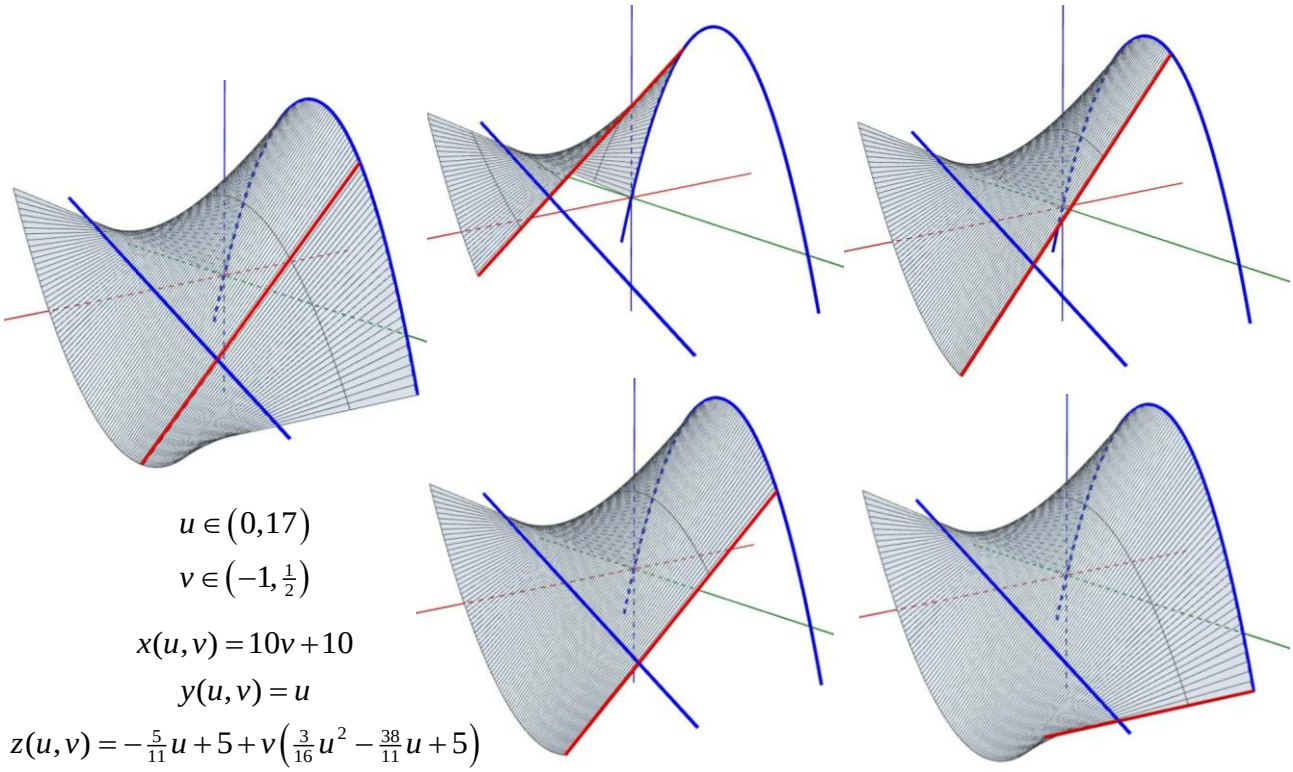

Figure 2: Example of the determination of a ruled surface using *GeoGebra*.

The typical task in descriptive geometry is to construct central or parallel projection (a two-dimensional image) of a surface or of its parts. An example of projections of a ruled surface is given in Figure 3. Firstly, the situation in the three-dimensional space is provided (i.e. 3D model), secondly two types of projections are shown - central and parallel projection of the same ruled surface (i.e. planar images). If we want to project complicated 3D object in parallel or central projections from general view point (i.e. we do not consider now simple top or front view of an object), we have to find a proper position in the three-dimensional space from which we can clearly observe this object. I use 3D modeling software for the determination this position and also for the automatic construction of the two-dimensional result of the projection. This is very useful tool because it is very difficult to estimate the proper view point of a surface manually. We can also use the determination of projection for testing purposes, i.e. the task for students is to construct the two-dimensional image of a surface in given projection from given inputs. Students can solve the tasks using 3D modeling software or they can draw the solutions by hand. It means that in this stage they work just in the plane. When using software, it is necessary to construct the silhouette of the surface; if drawn by hand, the aim is to depict some of the important curves on the surface. In both cases, the result is a planar image. An example of a geometric task is given in Figure 4, parallel projection and the determination of a ruled surface are given and the task is to construct an image of a surface (the result of the projection) and an intersection curve of a surface and a given cutting plane. 3D model of the constructed surface is also presented.

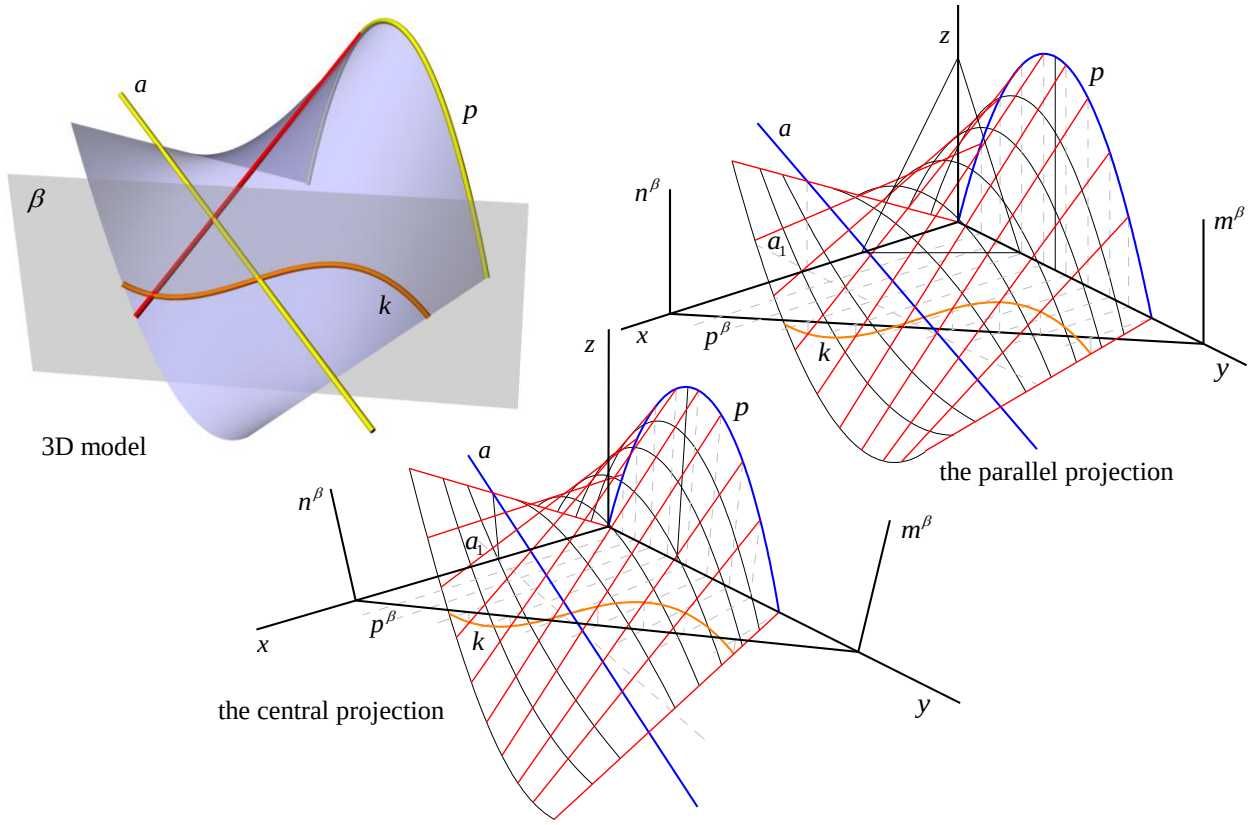

Figure 3: Example of projections of a ruled surface. The situation in the three-dimensional space, central and parallel projection of the same ruled surface.

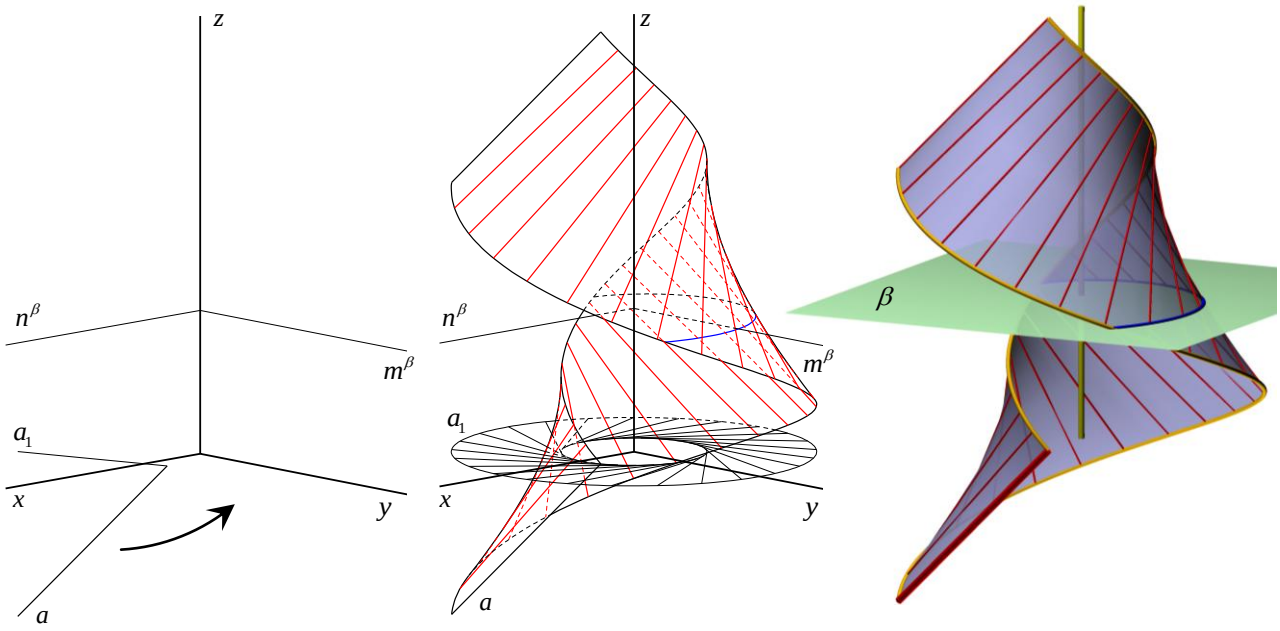

Figure 4: The determination of a parallel projection and a ruled surface. The result of the projection, an intersection curve of a surface and a given cutting plane, and 3D model of the spatial situation.

Students' work

My students meet 3D computer modeling within compulsory lessons of descriptive geometry and also can attend the seminars of applied descriptive geometry where practical applications of descriptive geometry and 3D modeling are mentioned and discussed. Students also use practically 3D modeling software during these lessons and seminars and can create themselves the outputs - 3D computer models and planar constructions. I am also supervising bachelor and master theses on various geometric topics where my students can use 3D modeling software. Figure 5 shows several examples of students' work.

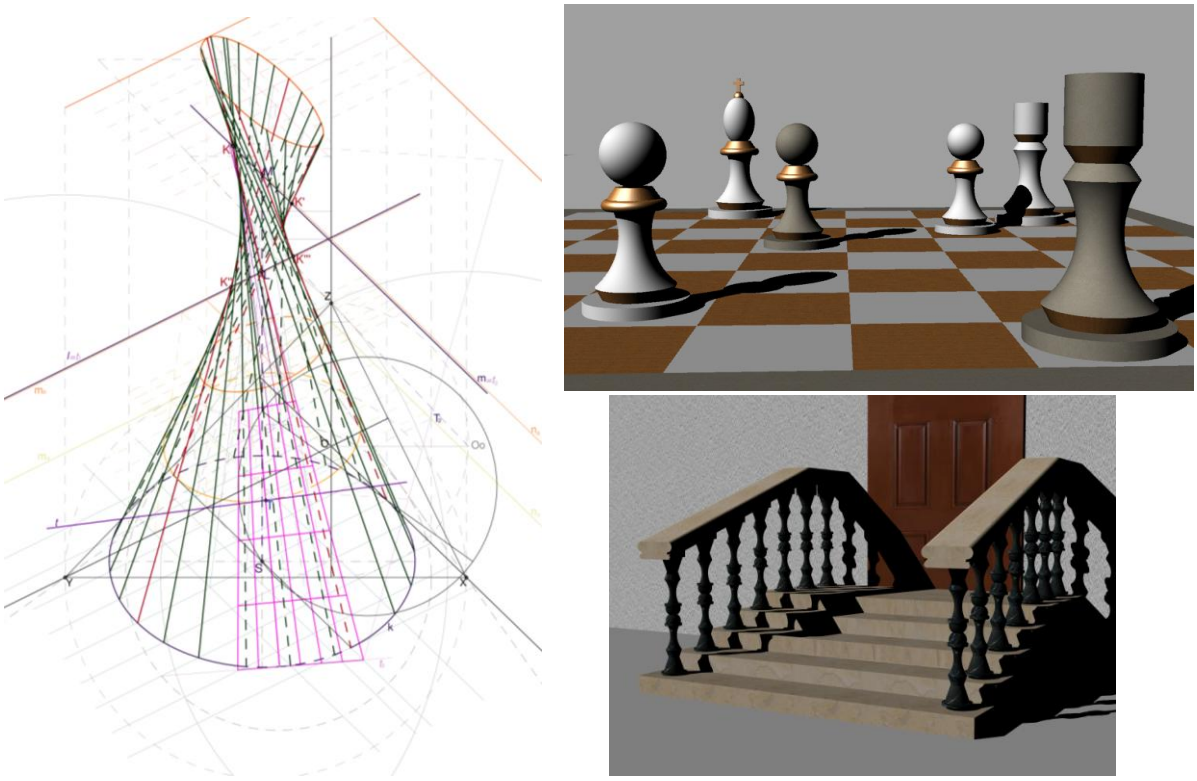

Figure 5: Students' work.

DISCUSSION, CONCLUSION, AND FUTURE WORK

I presented the possible methods of innovation in teaching descriptive geometry including 3D computer modeling and demonstrated this approach on examples of parallel and central projections of regular surfaces.

I have been teaching using described modern methods, activities and outputs for several years. When I started using it my aim was to minimize or even abandon the hand sketching and drawing. I had to change my mind very soon. According to my personal experiences computer aided classroom practice must be always accompanied by traditional explanation of geometry - i.e. hand sketching and drawing on the blackboard (lecturer and students) and on the sheets of paper (students) otherwise the students have the problems with taking notes during the lecture and understanding the individual steps of geometric constructions.

For instance, the projections which were shown in Figures 3 and 4, cannot be demonstrated to students only in this way. It is always necessary to show the construction of all lines, curves and points one by one. It even means that it is not enough when these objects just „appear" on the screen

one by one, students have to see the process of their drawings. In these situations is usually better to draw the constructions by hand on the blackboard because students can easily follow the instructor. The computer outputs can be shown at the same time.

3D computer modeling is an efficient aid for creating the animations as has been shown in Figure 2. These animations can be still accompanied with physical models on which we can demonstrate a sequential construction of geometric objects. According to my experiences there will be always students in a classroom who prefer more physical models which can touch by hand.

Regarding the future work I plan to focus on preparing a new textbook on descriptive geometry and to continue on the creation of study materials, 3D computer models, and another outputs for descriptive geometry. Besides these plans I would like to deal with the didactic survey and explorations which were introduced in this article, moreover to discuss following questions:

- advantages and disadvantages of computer-aided education of geometry (in general of mathematics),
- computer-aided education support and develop students' skills for future occupation in technical fields and branches,
- students' hand sketching and drawing (the both - in the classroom and during home preparation) is still necessary in currently digital era.

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