INTERACTIVE DIAGRAMS USED FOR COLLABORATIVE LEARNING

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The present research focuses on the development of knowledge about motion processes involving collaboration between students and interactive multiple representations diagrams. We designed three settings of interactive diagrams that share an example represented as an animation of multi-process motion but differ in their organizational functions. The 13- and 14-year-old students explored sets of characteristics of the mathematical models in the diagrams to analyze the related phenomena presented as real model and developed meaning of the abstract representations regarding the phenomena. The development of shared knowledge occurred when the students engaged in a reflective activity concerning the other members' reasoning and instruments involved in the collaborative process.

Keywords: Interactive Diagrams; Collaborative Learning; Mathematical Models; Animation

INTRODUCTION

Our research interests are concerned with the development of knowledge about mathematical models of motion processes involving collaboration between students and interactive diagrams. Interactive diagrams (IDs) are relatively small units of interactive text (in e-textbooks or other materials) and are important elements in e-textbooks. The ID`s components include: the given example, its representations (verbal, visual and other) and interactive tools. The difference between an ID and other interactive tools is that an ID is built around a pre-constructed example to carry a specific task. Whereas a static text presents information and a point of view implicitly engaging the viewer in meaningful interpretations, an ID explicitly requires the viewer to take action and change the diagram within given limitations.

Mathematical modeling is defined as the process of constructing a mathematical representation of reality that focuses on selected features of the reality being modeled (Cai, et al. 2014). To help learners construct mathematical representations of reality, the teaching-learning processes need to include the development of tools that will serve them in the practice. There are two approaches to teaching-learning mathematical modeling: (1) to learn by constructing models and (2) to learn by using models (Schwartz, 2007). But the two perspectives should not be in contrast with each other. Students who do not have experience with mathematical models will probably not benefit greatly from constructing their own models, if indeed they can learn to do so at all (*ibid.*). At first, learners tend to explore models by modifying their parameters. Next, they are often asked to modify the models themselves, thus providing them with the original and many similar models with which to work. Finally, students may be asked to devise models of phenomena independently. Pedagogic artistry, or the art of executing the teaching-learning process well, lies in helping students move through this sequence in ways that are appropriate to their current understanding of mathematical modeling.

Using technology to develop interactive curriculum materials, such as interactive textbooks, provides a captivating, engaging tool which encourages learners to explore mathematical models and to devise their own models as suggested by the learning sequence. The material presented in this way attempts to create new avenues for learners to develop knowledge about mathematical modeling. It is especially important that, while students learn about dynamic processes, such as motion, and about the mathematical models of the processes, the materials be represented in a similarly dynamic way by animations and interactive models in order to reinforce their knowledge development (Ainsworth, 2006;Yerushalmy and Naftaliev, 2011; Schwartz, 2007). The animations and models are simplifications that attempt to capture the essential features of the reality they describe. Technological developments are introduced into the range of resources available to students and teachers. In order to guide students to focus their attention on the essential details of the dynamic processes and to analyze the process, interactive curriculum materials should be designed to provide opportunities for exploration of mathematical models.

VISUAL SEMIOTIC ANALYSIS OF ID FUNCTIONS

There are profound differences between the traditional page in math textbooks that appears on paper and the new page that derives its principles of design and organization from the screen and the affordances of technology. Current technology allows for a variety of interactive tools, examples and representations. For example, IDs focused on motion may include the following components: a wide range of representations of motions; a wide repertoire of linking tools, and choices of activation of various representations. The question is, "How do a curriculum designer, a learner, and a teacher decide how and which IDs components of the text to use for different purposes in teaching-learning processes?" To explain some aspects of the design of an ID, we adopted a framework developed by semiotic research of text and visuals and provided a collection of categories that would allow an orderly discussion of the subject. (Naftliev and Yerushalmy, 2017; Yerushalmy, 2005). There are three ID's functions in the framework: the orientational function, the presentational function and the organizational function.

The presentational function focuses on what and how is being illustrated by the diagram. The reader may act within the context of the given example and change it or create other similar examples. Three types of examples are widely used: Random examples, Specific examples and Generic examples. The orientational function relates to the type of relationships that the text design attempts to set between the viewer and the text. IDs can function both as sketches and as diagrams in the sense that they can reveal their details.

The organizational function looks at the system of relations defining wholes and parts and specifically at how the elements of text combine. IDs can be designed to function in three different ways: Illustrating, Elaborating, Guiding. Illustrating IDs are simply operated unsophisticated representations. They are intended to orient the student's thinking to the structure and objectives of the activity by usually offering a single representation and relatively simple actions. For example, an Illustrating ID may have a limited degree of intervention by activation of controls in the animation (Table 1). At any time, users can freeze the positions on the track, continue the run, or initialize the race. Elaborating IDs provide the means that students may need to engage in activities that lead to the formulation of a solution and to operate at a meta-cognitive level. The important components in the design of the Elaborating IDs are rich tools and linked representations that enable various directions in the search for a solution. For example, the same animation that serves as an illustrating ID can be part of an elaborating ID when set within other tools and representations. The ID provides four adjacent, linked representations: a table of values that represents distance and time; a two dimensional graph of distance over time; a one dimensional graph which traces the objects' positions at each time unit; and an animation (Table 1). The variety of linked representations and rich tools in this elaborating ID enables various options in viewing the ID: as a sketch and/or as a neat diagram, as discrete information and/or as a continuous flow of information. We use the term Guiding IDs in relation to guided inquiry. This kind of diagram provides the means for students to explore new ideas. In addition to providing resources that promote inquiry, they also set the boundaries and provide a framework for the process of working with the task. The Guiding IDs are designed to call for action in a specific way that supports the construction of the principal ideas of the activity and may serve to balance constraints and open-ended explorations and support autonomous inquiry. For example, the guiding ID was designed around a known conflict about a time-position graph describing a "motionless" situation over continuously running time (Table 1). The ID consists of two representations of the motion of four cars: an animation and a hot-linked position-time graph. The task is to establish a one-to-one correspondence between the graphs and the cars. The graph and the animation are only partially linked: motion occurs simultaneously on the animation and on the graph but there is no color-match, so the identification process requires extracting data from the animation and the graph in order to link them. The following constraints contribute to making the task an interesting challenge: the small number of animated representations, the partial link between the representations, the absence of representations and controls that could turn the given sketchy nature of the representations into an accurate diagram, and the exceptional example in a list of examples that are aimed at focusing on a motionless situation over time.

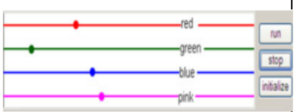
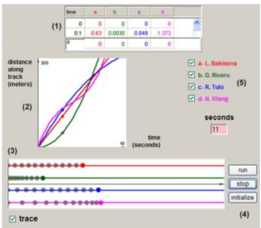
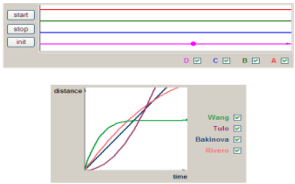
	Illustrating ID 	Elaborating ID 	Guiding ID 
Animation "run“, "stop", “timer” choose components	 ✓, ✓, × ✓	 ✓ , ✓, ✓ ✓	 ✓ , ✓, × ✓
1D graph	×	✓	×
2D graph	×	✓ (neat and sketch)	✓ (sketch)
Table of Values	×	✓	×
Links between representations	×	✓	✓ (partial)
Examples	Generic	Generic	Generic (motionless component)

Table 1. Comparative view on the IDs’ design

RESEARCH DESIGN

The series of activities included a preliminary activity and three comparable activities that contained different ID's. The IDs shared an example represented as an animation of multi-process motion but they were different in their organizational functions. The activity, which asked the students to describe a motion situation, was first illustrated by a video clip and subsequently as an Illustrating, Elaborating or Guiding IDs, all based on the animation. The three IDs varied by the design choices concerning what was included in the given example and how it was represented and controlled.

Regarding what to include in the example, the animation was designed around simultaneous multi-process motions, to include motion situations known to be challenging, such as non-constant rate-of-change and "no motion" situations, as well as surprising situations such as an ”unexpected win“. Considerations of how to design these choices were driven by the semiotic functions framework. We made comparative decisions about the variety and type of representations, the control features, and the linking features.

The interviews were sorted by students into groups of three. Each interviewee met the interviewer twice. The first meeting included an interview with each student individually. The second meeting was a group interview. Each participant followed a three-step procedure that enabled us to examine and track the role of IDs in the students' knowledge development process concerning mathematical models of motion. At the first stage, the students were given a preliminary task presented as a video clip and designed to evaluate their knowledge and solution techniques. At the second stage, the students were given a task similar to the one they received before, except that it was presented as an ID. The purpose of the interview was to learn how the students constructed their knowledge using the diagram. At the third stage, the three students who had been asked to address similar tasks that included different IDs shared their work and participated in a group discussion. The students were asked to describe the technique they used in their solution, to present their use of the ID, to reflect upon their changes and to be involved in a conversation regarding other students' techniques. The students could use all the diagrams they worked with in the previous stages. The interview time was flexible and varied according to the participants' responses. All the interviews were video recorded.

For the first step of the research, we analyzed the students' emerging, personal, engagement processes as they interacted with one of the three mathematical modeling IDs which were designed to support different functions of inquiry teaching-learning. The findings of our previous research show that similar tasks with different IDs should be considered as different learning settings (Naftaliev and Yerushalmy, 2013, 2017; Yerushalmy and Naftaliev, 2011). In the presented research, we focus on the second step by asking the students who had already been asked to address similar activities that included different IDs to share their work and to participate in a group discussion. The process allowed us to analyze the social construction of knowledge in a new pedagogical setting.

THE SOCIAL CONSTRUCTION OF KNOWLEDGE IN A NEW PEDAGOGICAL SETTING

We are going to analyze one of the groups' engagement processes to present the social construction of knowledge in a new pedagogical setting (Table 2). The group has one student for each of the three types of IDs: Illustrating, Elaborating, and Guiding. We'll begin by looking at their individual work in the two first stages. With video clips, the learners put the emphasis on getting the story right, which required attending to details such as the runners' body motion. The video clip kept learners too close to the situation and prevented them from thinking in the abstract. Elad, the student who worked with the illustrating ID, started by activating the animation. Throughout the process, he stopped the animation several times. During each pause, Elad examined the runners' respective positions and described the changes in speed between each stop. Elad described each runners' changes in speed with reference to their relative positions at specific moments. He mistakenly interpreted continuous change of speed by comparing relative positions. For example, he argued that passing another runner must have meant speeding up; whereas, in reality, the runner maintained a constant speed. To cope with the challenge, Elad resorted to a failed attempt at drawing graphs by himself to complete the diagram. Helena, the student who worked with the elaborating diagram, started by activating the representation and tools in the ID. She learned about the wide variety of options and representations available in the ID, but we didn't have evidence that showed developing knowledge concerning mathematical models of motion processes. Or, the student who worked with the Guiding ID, began his work by identifying a visual and kinematic conflict: while all seven dots moved on the graphs, one of the dots in the animation stopped and remained still. To resolve this conflict, he focused on discrete events much like Elad, using discrete events to match the motions described in the animation and graph extracting discrete motion characteristics such as: average speed, time and distance. He successfully matched the dots yet failed to resolve the conflict.

Knowledge development concerning Characteristics of Motion and the elements of IDs	In stage 2			In stage 3
	Elad with Illustrating ID	Helena with Elaborating ID	Or with Guiding ID	Elad, Helena and Or
Familiarize him/herself with the elements of the IDs	✓	✓	✓	✓
Discrete Characteristics (Animation): average speed, time for distance, distance	✓		✓	✓
Discrete Characteristics (Graph): average speed, time for distance, distance			✓	✓
Continues Characteristics describing the motion process (Animation and Graphs): such as speed, time, and distance as variables in motion processes				✓

Table 2. Knowledge developments in the second and third stage

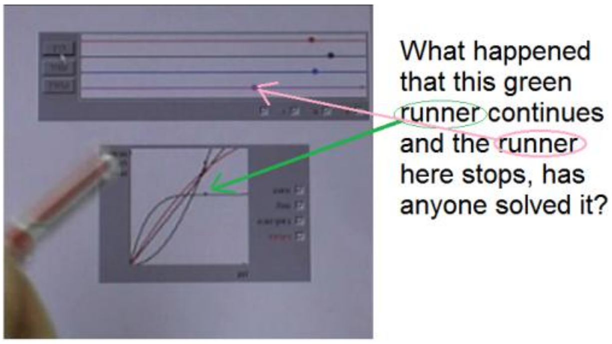


Fig. 1 "…has anyone solved it?"

Progressing to the group discussion, Or decided to open the conversation with the question which remained unsolved in his individual work (Fig. 1). He demonstrated the problem while activating the Guiding ID with which he worked. The two other participants, Elad and Helena, were intrigued by the question and it turned into the goal of their collaborative work. They began by familiarizing themselves with the options of the ID and examples presented in it to resolve the conflict. When they didn't have success resolving the conflict using the Guiding ID and realized their diagrams were different, Helena suggested using representations and tools from her Elaborating ID to accomplish the goal they defined for themselves. Each time she suggested adding only one of the options from the Elaborating ID. They used it firstly to develop meaning regarding the motion presented in the Elaborating ID. Then, they used the ideas which they developed to resolve the conflict using the Guiding ID. The following dialogue presents the process which took place in the last step of their work in which they successfully resolved the conflict.

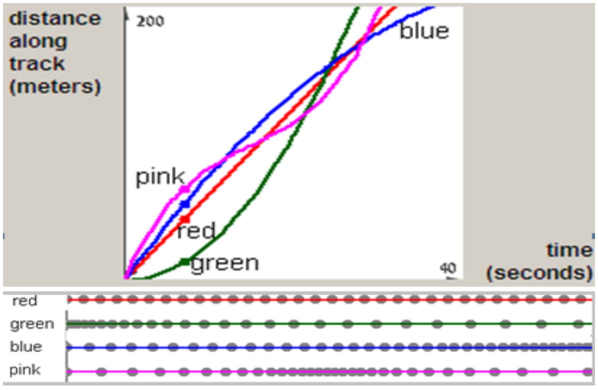


Fig. 2 2D and 1D (traces) graphs recorded while running the animation

Following suggestion of Helena, the students activated the animation with traces, resulting in the generation of a 1D graph of the motion (Fig. 2). While running the animation and generating of a 1D graph, they read the race from the traced motion using the size of the spaces between the traces as a gauge for speed:

Helena: Press on traces. You see! Where they are stopping?

Or: Ahh… Yes, it describes every time point.

Elad: It describes the steps, the distance of the steps.

Helena: Here, you see the black starts [green] to advance more

Elad: Pink starts with greater steps. If the traces describe the steps then here he starts to slow down as the time goes on and here it stays at the same speed

Helena: And the black [green] is really fast

Elad: But in the end he speeds a bit. The black [green] almost doesn't, he starts with slowness, as the time goes on, his steps only enlarge

Helena: The red doesn't change… and the red.. At the same speed

Elad: And the red, like I told you in the beginning, remember? That the red is always at the same distance, at the same speed, the same steps. And the blue at the beginning until the middle at the same speed, same steps and towards the end he starts to slow down.

Following the interpretation of the 1D graph as describing speed, the students check whether this option is available in the Guiding ID. Once they verify it is not, they returned to work with Elaborating ID. They began by interpreting the 2D graph based on the 1D graph in static mode with which they became familiar. At the end they were able to describe the speed by using only the 2D graph.

Helena: Wait, in his [Or] diagram there is it [the traces]? Check

Or: Check

Helena: It's interesting what happened with the pink in his [Or] diagram

Or: No. I think that this [the elaborating ID] is the best.

Helena: The red is running at the same speed. The black in the beginning runs really slow, and then he ups his speed more and more [they closed the 1D graph and continued work only with the 2D graph]. The blue runs really quick and then he starts to slow down. The pink runs fast, in the middle he slows down and then in the end again he runs fast.

Once they have succeeded in interpreting the 2D graph in the Elaborating ID, they were able to resolve the conflict they had about motionless process presented by the Guiding ID:

Or: Yes. So, as the line is steeper, then his speed is... ehh... it is steep and… that's it, I see that in the end it turns into a straight line, plain, something like this. That means that he slowed the speed and even stopped in place.

Elad: If this shows distance, then it means that the distance here does not change.

The episode describes exploration concerning the speed description in four stages: analysis of a dynamic mode of 1D graph which was linked to running animation, analysis of a static representation of 1D graph, analysis of shapes of 2D graphs and analysis of motionless process represented by 2D graph. They examined the graphs, each being composed of curving segments, representing an increase or decrease in speed, and straight segments, describing constant speed.

DISCUSSION

Students do need to have enough experience with abstract models to understand the point of mathematical modeling, its "language" (Schwartz, 2007). Once such representations exist in cognitive "baggage" of learners," it also becomes a tool for mathematical modeling (Wilensky, 1999). In our research we focused on knowledge development concerning mathematical models involving collaboration between students and various IDs.

The students explored sets of characteristics of the mathematical models in the IDs to analyze the related phenomena presented as real model and developed meaning of the abstract representations regarding the phenomena. They looked for ways to bypass the designed constrains of the Guiding ID: they develop meaning regarding the motion represented in the real and mathematical models by using the Elaborating ID, pointing to the speed, time, and distance as continues variables. Then they used the ideas which they had developed to analyze the characteristics of motion presented in the Guiding ID. At the end of the discussion, the mathematical models in static mode prompted them to mentally recreate and describe the motion processes.

The development of shared knowledge occurred when the students engaged in a reflective activity concerning the other members' reasoning and instruments involved in the collaborative process. As a result of the group collaboration, the students generated an interactive text. The participants did the following: posed a new question, decided what component from what ID to bring to discussion, decided on the sequence between the components, defined the role of each component, and, created a representation of the data. All of this was done to accomplish the goal they posed to themselves, thus building meaning concerning mathematical models of motion. Ainsworth (2006), who investigated the use of various technological packages that provide similar pairs of representations for understanding the mathematics of motion, suggested that the second more familiar or concrete representation was intended to bridge understanding of the more complicated and unfamiliar representation. Our analysis clarified that choosing and combining representations from similar tasks, which were designed as different IDs, reflected students' personal choices to anchor their inquiry in the ones they noticed first or with which they were more familiar. The interactive texts became an instrument which supported the development of shared knowledge concerning characteristics of kinematic phenomena and about their mathematical models.

IMPLICATIONS

To educators, who are challenged by the design and the implementation of interactive mathematics instructional materials, this study offers ways and terms to think about the design of interactive texts. Teaching with an interactive textbook should be considered more than a technological change; indeed, it is an attempt to create new paths for the construction of mathematical meaning. Other considerations related to specific affordances of technology should be studied to make the currently offered technological shift in learning and teaching materials an important sustained pedagogical shift.

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