CAN I SKETCH A GRAPH BASED ON A GIVEN SITUATION? – DEVELOPING A DIGITAL TOOL FOR FORMATIVE SELF-ASSESSMENT

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This article describes the development of a digital tool for formative self-assessment in a design-based research study. The aim is to create a tool that allows students to self assess their work, rather than having technology evaluate their answers. Thus, learners are provided with a list of typical misconceptions to check their solutions to an open assessment task. This assessment task tests the students' ability to draw a graph based on a given situation. Two case studies in form of task-based interviews with sixteen-year-old students are described. The analysis leads to reconstruction of the learners' formative assessment processes by using a theoretical framework developed in the EU-project FaSMEd. The results show which formative assessment strategies students actively use when working with the digital tool and which functionalities of the technology can be identified.

Keywords: formative self-assessment, role of technology, functions, design-based research

AIM OF THE TOOL

A challenge for the design of a digital tool for student formative self-assessment is that the actual assessment should not be done by the technology. Some digital self-assessment environments generate a set of questions, check the student's answers based on two categories: right or wrong; and then provide the student with feedback in form of the number of correct responses. However, while a student works individually in such environments, he/she does not adopt the role of the assessor. Therefore, the term "self"-assessment refers only to the organisation of the assessment for such tools. In order to move the learning process forward, it is essential for the student to gain information on his/her own understanding of the learning content (Wiliam & Thompson, 2008). Moreover, the active involvement of learners is identified as a common characteristic of effective formative assessment approaches. Investigating their (mis-)conceptions helps students to gain sensitivity for their strengths and weaknesses. In addition, students can discover how to observe and direct their learning processes using metacognitive strategies along with reflection and adopt responsibility for their own learning in the process (Black & Wiliam, 2009; Heritage, 2007). Hence, a key design feature of our tool is a checklist of typical misconceptions related to the mathematical content, which is the change from a situational to a graphical representation of a function, that helps students to become self-assessors. The tool was developed during the design-based research EU-project FaSMEd (Raising Achievement through Formative Assessment in Science and Mathematics Education), which introduced and investigated technology enhanced formative assessment practices (www.fasmed.eu).

THEORETICAL BACKGROUND

Conceptualising formative assessment

Formative assessment (FA) is "the process used by teachers and students to recognize and respond to student learning in order to enhance that learning, during the learning." (Bell & Cowie, 2001, p. 540). It results in the active adaptation of classroom practices to fit students' needs by continuously gathering, interpreting and using evidence about ongoing learning processes (Black & Wiliam, 2009). The required data can be elicited and exploited during the different phases of these processes. Wiliam and Thompson (2008) refer to Ramaprasad (1983) and focus on three central steps in teaching and learning, namely establishing: where the learners are, where the learners are going and how they might get there. The authors state that FA can be conceptualised in five key strategies (Figure 1).

These strategies enable teachers, peers and students to close the gap between the students' current understanding and the intended learning goals.

	Where the learner is going	Where the learner is right now	How to get there
Teacher	**1** Clarifying learning intentions and criteria for success	**2** Engineering effective classroom discussions and other learning tasks that elicit evidence of student understanding	**3** Providing feedback that moves learners forward
Peer	Understanding and sharing learning intentions and criteria for success	**4** Activating students as instructional resources for one another	
Learner	Understanding learning intentions and criteria for success	**5** Activating students as the owners of their own learning	

Figure 1: Key strategies of FA (Wiliam & Thompson, 2008)

While Wiliam and Thompson (2008) take into account central steps of the learning process and the agents (teacher, peers and learners) who act in the classroom, their framework regards mainly the teacher to be responsible for the process of FA. It is the teacher who creates learning environments to investigate the students' understanding (strategy 2), who gives feedback (strategy 3) and who activates students as resources for one another (strategy 4) and as owners of their own learning (strategy 5). In order to regard all three agents as being able to take responsibility for each of the steps and key strategies, the framework was refined in the FaSMEd project. The FaSMEd framework (Figure 2) allows the characterisation and analysis of technology enhanced FA processes in three dimensions: agent/s, FA strategies and functionalities of technology (www.fasmed.eu; Aldon, Cusi, Morselli, Panero, & Sabena, 2017).

The "agent/s" dimension specifies who is assessing: the student, peer/s, or the teacher. It is important to involve all of the agents in FA as the "assessment activity can help learning if it provides information that teachers and their students can use as feedback in assessing themselves and one another […]" (Black, Harrison, Lee, Marshall & Wiliam, 2004, p.10). Moreover, an active involvement of students by peer and self-assessment is stated as key aspect of FA. It includes opportunities for learners to recognize, reflect upon and react to their own/ their peers' work. This helps them to use metacognitive strategies, interact with multiple approaches to reach a solution and adapt responsibility for their own learning (Black & Wiliam, 2009; Sadler, 1989).

The "FA strategies" dimension of the FaSMEd framework refers to the five key strategies (Wiliam & Thompson, 2008), but understands them in a broader sense by acknowledging that all agents can be responsible for FA. For example, a student can elicit evidence on his/her own understanding (strategy 2) by working and reflecting on assessment tasks, peers can provide effective feedback (strategy 3), or a student can control his/her own learning process using metacognitive activities (strategy 5).

To specify the different functionalities that technology can resume in FA processes, FaSMEd introduced a third dimension to the framework: "functionalities of technology". We distinguish three categories:

(1) *Sending & Displaying*, which includes all technologies that support communication by enabling an easy exchange of files and data. For example, the teacher

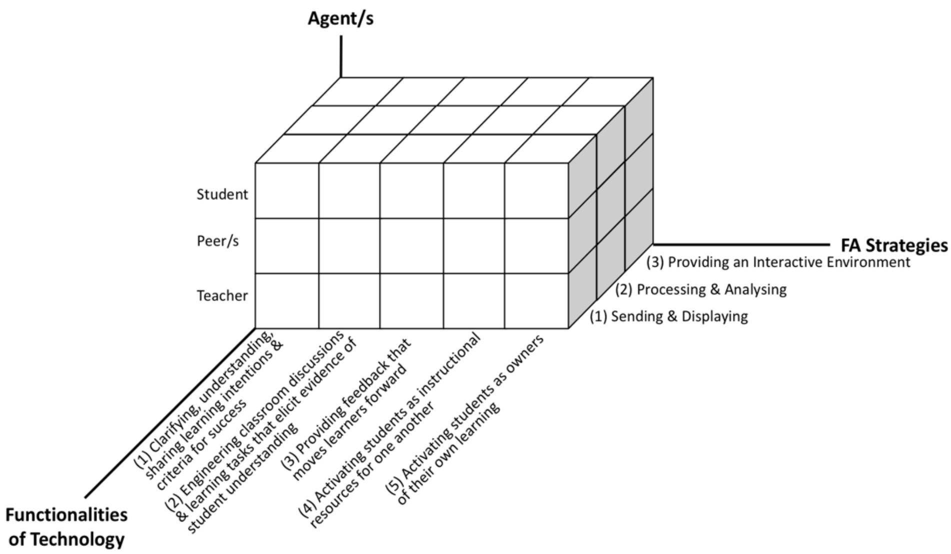

Figure 2: The FaSMEd framework

sending questions to individual students' devices or displaying one student's screen to discuss his/her work with the whole class.

(2) *Processing & Analysing* considers technology converting collected data. This includes software that generates feedback and results to an operation or applications which create statistical diagrams of a whole class' solution, for example after a poll.

(3) *Providing an Interactive Environment* refers to technology that enables students to work in a digital environment and lets them explore mathematical or scientific contents interactively. This category includes, for example, shared worksheets, Geogebra files, graph plotting tools, spread sheets or dynamic representations (www.fasmed.eu).

The mathematical content: Functions

During the development of a self-assessment tool, its mathematical content needs careful consideration. Bennett (2011) states that "to realise maximum benefit from formative assessment, new development should focus on conceptualising well-specified approaches […] rooted within specific content domains" (p.5). Therefore, a content analysis needs to evaluate, for example, which competencies or skills students need to master, what a successful performance entails and which conceptual difficulties might occur. This 'a priori' analysis revealed three aspects relating to functions relevant for the tool's development: different mental models that students need to acquire for a comprehensive understanding, translating between mathematical representations and known misconceptions.

The German tradition of subject-matter didactics specifies the idea of mental models in the concept of 'Grundvorstellungen' (GVs). It is used to "characterize mathematical concepts or procedures and their possible interpretations in real-life" (vom Hofe & Blum, 2016, p.230). Thereby, GVs identify different approaches to a content that makes it accessible for students. They describe, which mental models learners have to construct in order to use a mathematical object for describing real-life situations. In this sense, GVs act as mediators between mathematics, reality and the learners' own conceptions (vom Hofe & Blum, 2016). When using the graph of a function to describe a given situation, students have to acquire three GVs for the concept of functions: mapping, covariation and object. In a static view, a function maps one value of an independent quantity to exactly one value of a dependent quantity. The graph of a function can, thus, be seen as a collection of points that originate from uniquely mapping values of one quantity to another. In a more dynamic view, a function describes how two quantities change with each other. Considering a functional relation with this focus allows a graph to embody the simultaneous variation of two quantities. Finally, a function can be seen as a whole new mathematical object. Then, the graph is viewed from a global perspective (Vollrath, 1989).

Besides constructing these three GVs, a comprehensive understanding of the concept requires students to be able to change between different forms of representations of a function (Duval, 1999). Functional relations appear in a range of semiotic representations. Learners encounter them, for instance as situational descriptions, numerical tables or Cartesian graphs. Each of these emphasizes different characteristics of the represented function. Thus, transforming one form into another makes other properties of the same mathematical object explicit (Duval, 1999). What is more, Duval (1999) stresses that mathematical objects are only accessible through their semiotic representations. Therefore, each mathematical activity can be described as a transformation of representations. Duval (1999) differs between treatments, meaning the manipulation within the same semiotic system, and conversions, meaning the change of one representational register to another while preserving the meaning of the initial representation. The author identifies conversions between different registers to be the "threshold of mathematical comprehension for learners […]" (Duval, 2006, p.128) and concludes that "only students who can perform register change do not confuse a mathematical object with its representation and they can transfer their mathematical knowledge to other contexts different from the one of learning" (Duval, 1999, p.10). Hence, asking students to draw a graph based on a given situation means assessing a key aspect of their understanding of the concept of functions.

As students' mistakes can mirror their conceptual difficulties, typical misconceptions in the field of functions are considered for the development of our digital self-assessment tool. For instance, Clement (1985) states that many students falsely treat the graph of a function as a literal picture of the underlying situation. They use an iconic interpretation of the whole graph or one of its specific features instead of viewing it as an abstract representation of the described functional relation (Clement, 1985). To overcome this mistake, students need opportunities to consider graphs symbolically. Thus, instructions might ask learners to interpret a graph point by point or to describe the change of the dependent quantity for certain intervals. Another example of a typical cognitive issue when graphing functions is the 'swap of axes' labels. This mistake can arise when students name the axes intuitively without regarding mathematical conventions (Busch, 2015). Hadjidemetriou and Williams (2002) even speak of the "pupils' tendency to reverse the x and the y co-ordinates" (p.4). In order to correctly label the axes for a given situation, learners need to understand the functional relation between two quantities from its description and apply the convention to record the independent quantity on the x-axis and the dependent one on the y-axis of a Cartesian coordinate system (Busch, 2015). These are examples of some of the findings on typical misconceptions that were used in the design of our tool that both anticipate certain student difficulties and provide hints to foster the desired competencies.

DESIGN OF THE DIGITAL SELF-ASSESSMENT TOOL

The structure of the tool draws on a set of self-assessment materials originating from the KOSIMA (German acronym for: contexts for meaningful mathematics lessons) project (Barzel, Prediger, Leuders & Hußmann, 2011). Therefore, the tool comprises five parts: *Test*, *Check*, *Info*, *Practice* and *Expand*. These are connected in a hyperlink structure and labelled with different symbols (Figure 3) to support easy learner orientation regarding the tool's use.

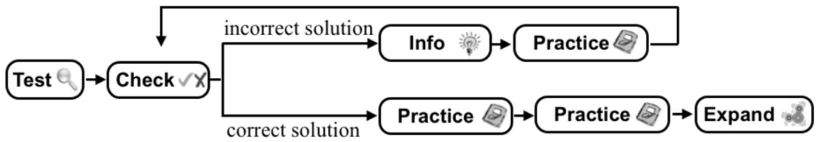

Figure 3: Hyperlink structure of the digital self-assessment tool

The aim is to create a tool that allows students to be self-assessors, that is why the design intends to create a balance between providing enough information as well as autonomy for the learners. The initial step of the self-assessment process is for the student to identify the learning goal. It is specified and made transparent in our tool by the question: "Can I sketch a graph based on a given situation?", which appears on the top of the first screen (Figure 4). The learner is provided with the *Test* task (labelled with a magnifying glass icon). This *Test* presents the story of a boy's bike ride and asks the student to build a graph that shows how the boy's speed changes as a function of the time. Besides labelling the axes by selecting an option from drop-down menus, the learner can build his/her graph out of moveable and adjustable graph segments. These are dragged into the graphing window and placed in any order the student chooses. Furthermore, the slope of the single segments can be altered by the user. After submitting a graph, a sample solution and *Check* are presented to help evaluate the individual answer (Figure 4). The *Check* is labelled with the symbol of a positive and negative check mark. It presents the student with six statements regarding important aspects of the functional relation at hand alongside common mistakes that could arise when solving the *Test* task. For example, one of the *Check*-points addresses the graph's slope: "I realized when the graph is increasing, decreasing or remaining constant.", or another represents the Graph-as-a-picture mistake: "I realized that the graph does not look like the street and the hill." The learner decides for each statement, if it is true for his/her solution, in which case it is marked off. For this diagnostic step, the student's screen not only presents the *Check*-list, but his/her answer as well as a sample solution to make a comparison easy. Thus, the *Check* helps the learner to self-assess his/her solution by presenting criteria for successfully solving the *Test* and by encouraging reflection of one's answer in comparison to the sample solution and *Check*-points. Additionally, the *Check* serves as a directory through the tool's hyperlink structure (Figure 3). This way, the student is encouraged to take further steps to move his/her learning forward.

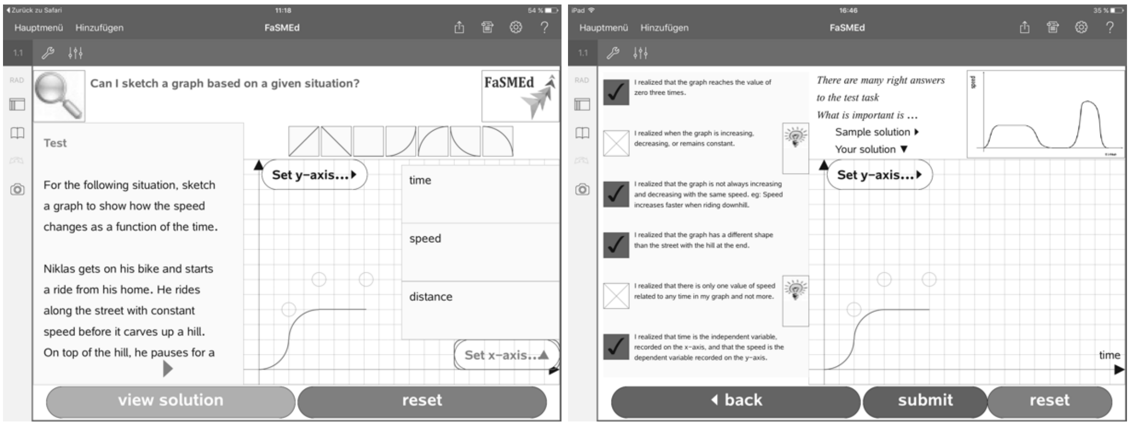

Figure 4: *Test* and *Check* of the digital self-assessment tool

If an error is identified by the learner, he/she can choose to work on the *Info* and *Practice* task corresponding with the *Check*-point's statement. The *Info* is labelled by the symbol of a lightning bulb. It entails a general explanation that is intended to repeat basic classroom contents to overcome the certain mistake. Moreover, the explanation is made accessible by using the time-speed context of the *Test* as an example. In addition, an illustration is included to ensure a visual help and to encourage the learner to change between the two semiotic representations: verbal description as well as Cartesian graph. Then, the *Practice* task lets the student test his/her understanding of the repeated content. It is marked by the picture of an exercise book. Afterwards, the user can go back to the *Check* and work on the next statement. If the sketched graph is stated as correct, two further *Practice* tasks and one *Expand* task with a more complex context are provided. The *Expand* is labelled with a gearwheels icon and, in this case, asks the student to draw two different graphs for the same situation.

Above all, the tool aims to challenge the student to reflect on his/her own solutions and reasoning. This is why, besides offering a *Check*-list, it presents sample solutions for all tasks. It is the learner who decides weather the own answer is correct by comparing it to the sample solution.

METHODOLOGY

The conception and evaluation of the digital self-assessment tool are connected within a design-based research study. This is a "formative approach to research, in which a product or process is envisaged, designed, developed, and refined through cycles of enactment, observation, analysis, and redesign, with systematic feedback from end users" (Swan, 2014, p.148). Here, two different forms of case studies are applied: class trials and student interviews. The purpose of the class trials is to evaluate the effectiveness of the tool's implementation by exploring whether: self-assessment is possible using the tool, the structure is clear, and any technical issues are identified. Hence, class trials are conducted during a lesson where students work on the digital self-assessment tool individually or in pairs. Data is collected in the form of the researcher's notes on the lesson and a classroom discussion about the students' experiences with the tool. In addition, task-based interviews with individual students aim for a more detailed understanding of the learners' FA processes. This is why, students are asked to "think out loud" during their work with the tool and interviewers are instructed to only intervene the students' self-assessment to remind them to verbalise their thoughts or to help with technical issues. At the end, reflecting questions about the students' experience with the tool are asked. The interviews are videoed and transcribed to serve as the main data pool for qualitative analyses. These lead to the reconstruction of FA processes using the FaSMEd framework (Figure 2). Besides generating a well-grounded tool, the aim of the study is to examine the following research questions:

When students work with the digital self-assessment tool:

1) which formative assessment strategies do they use?
2) which functionalities does the technology have within the student's FA processes?

In each cycle of development, the investigation of these questions using the FaSMEd framework (Figure 2) informs the re-design of the tool. On this account, several development cycles took place in the study since 2014. A first pen-and-paper version of the tool was evaluated through interviews with eleven grade eight students from two different secondary schools in Germany.

Following the tool's redevelopment, two digital prototypes were created using different technologies: JACK and TI-Nspire Navigator. JACK is a server-based system for online assessment developed by the Ruhr Institute for Software Technology at the University of Duisburg-Essen. While the software has several useful options, such as being able to generate automatic feedback based on student answers, to create statistical overviews of submitted solutions and to insert tasks with variable contents, the JACK prototype proved to be unfit for implementation of our tool due to three main reasons: First, its hyperlink structure could only be implemented in a restricted way. It was not possible to display the entire *Check*-list at once, but only single *Check*-points. Furthermore, the software has a limited number of task types that are mainly in form of multiple choice or open answer formats. Finally, JACK requires an internet connection, but most schools in Germany do not have access to wireless internet in their classrooms, which would limit its potential use. The second digital prototype was programmed in Lua script using the software TI-Nspire Navigator, which enabled the tool's hyperlink structure to be realized, offline access and a choice of using the tool on a computer or iPad. Moreover, the options for implementing open tasks were greater and dynamic visualisations could be inserted. Hence, the tool's design was implemented only for TI-Nspire Navigator. The subsequent classroom trial of the digital tool run on iPads involving 18 grade ten students led to further redevelopments.

The finished digital version was trialled in two grade ten classrooms at two further secondary schools and associated student interviews (one per class) were recorded. Finally, another set of student interviews with two second semester university students were held. The wide range of data in different age groups and schools resulted in a thorough evaluation of the tool's potential and constraints. As it is intended to assess and repeat basic mathematical competencies, its use is not limited to one specific group of learners. First experiences with the tool show that students in all of the tested class levels (grades 8, 10 and university) had similar issues concerning mathematical understanding as well as technical problems. This article focuses on the two single student interviews recorded in grade ten.

RESULTS

Two students' work with the digital tool are presented and their FA processes analysed using the FaSMEd framework (Figure 2). Both learners (S1 & S2) are female and sixteen years old, but visit different secondary schools. Their interviews were chosen for the analysis because they both trialled the digital version of the tool and selected the same *Check*-point regarding switching the x- and y-axis labels to take further steps in their learning. Both students start with the *Test* task (see text box).

> *For the following situation, sketch a graph to show how the speed changes as function of the time.*
>
> *Niklas gets on his bike and starts a ride from his home. He rides along the street with constant speed before it carves up a hill. On top of the hill, he pauses for a few minutes to enjoy the view. After that he drives back down and stops at the bottom of the hill.*

S1 built her graph (Figure 5) by dragging moveable graph segments into the graphing window and selecting labels for both axes from drop-down menus. As she solved the assessment task, she evidences her understanding of sketching graphs of given situations (strategy 2) while the tool provides an interactive learning environment (functionality 3). After reading the sample solution out loud, S1 moved to the *Check* and was silent for a while. The interviewer asked what she was thinking about. The student mentioned being unsure about which *Check*-list items to mark off because she "saw in the sample solution that there was another graph and this was missing in [her] own solution." With the "other graph" she means a second hill-shaped part of the graph, which she indicated by gesturing its shape on the screen with her finger. It can be concluded that the *Check* stimulates S1 to assess her answer by comparing her own graph to the sample solution. By reflecting on her answer, S1 uses a metacognitive activity and, thus, adopts some responsibility for her own learning process (strategy 5). The tool displays the information she needs for the diagnostic step in form of the sample solution and *Check*-list (functionality 1). Furthermore, the student decided to evaluate the last statement in the *Check*. It reads *"I realized that the time is the independent variable recorded on the x-axis and that the speed is the dependent variable recorded on the y-axis."* S1 stated that this was not true for her graph, which means that she understands a criterion to successfully solve the *Test* (strategy 1). What is more, she reflects on her solution by comparing it to the *Check*-point statement (strategy 5) and formulates a self-feedback (strategy 3): "The speed and time were wrong because there [she points to x-axis] needs to be the time and there [she points to y-axis] the speed. I did not realize this." Here, the technology is once more functioning as a display of information in the form of the *Check*-point (functionality1).

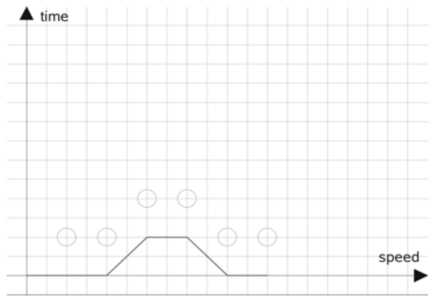

Figure 5: S1's *Test* solution

At that point S1 decided a next step in her learning (strategy 5) when she read the associated *Info*. After the interviewer reminded her of the possibility to do another exercise related to her mistake, S1 worked on the linked *Practice*. This helps her to elicit evidence about her understanding of the independent and dependent quantity of a functional relation (strategy 2). The tool provides the task and sample solution (functionality 1). The task presented the learner with ten different situations describing the functional relation between two quantities. For each one, the learner was asked to assign labels to the axes of a coordinate system (given that he/she imagined drawing a graph based on the situation in the next step). The labels were chosen from a number of given quantities: temperature, distance, speed, time, pressure, concentration, money, and weight. S1 solved six out of ten items correctly. While she seemed to have no difficulties with situations in which time appeared as the independent quantity, she struggled to label the y-axis when time was being dependent on another quantity. For example, in the situation *"In a prepaid contract for cell phones, the time left to make calls depends on the balance (prepaid)."* S1 chose "time" as the label for the x-axis and "money" as the label for the y-axis. However, she explained "if you have a prepaid phone, you can only make calls as long as you have money." Therefore, she grasped the relation in the real-life context but ccouldn't use this knowledge when asked to represent it in form of a graph. Moreover, the student repeated this mistake of 'swapping the axes' even in situations that didn't include time as a quantity. For instance, S1 selected "distance" as the label for the x-axis and "speed" for the y-axis in the situation *"Tim's running speed determines the distance he can travel within half an hour."* Nonetheless, she explained correctly that "the speed specifies how far he can run." A possible explanation for her repeating mistake could be her approach to the task. S1 selected a label for the y-axis first before going on to the x-axis. This could mean that she does not fully understand the conventions of drawing a Cartesian coordinate system. However, her mistake could also originate from a deeper misunderstanding as Hadjidemetriou and Williams (2002) speak of the "pupils' tendency to reverse the x and the y coordinates" and their inability to adjust their knowledge in unfamiliar situations" (p.4). This would show a need for further interventions. However, S1 was able to identify two out of her four mistakes by comparing her answers to the sample solution (strategy 5) before she returned to the *Check* and marked off the respective *Check*-point statement.

In summary, S1's work with the digital self-assessment tool can be depicted as shown in Figure 6. She solves a diagnostic task, identifies a mistake by understanding criteria for success, reflecting on her answer and comparing it to a sample solution and displayed statement. She gives herself feedback and decides to take further steps in her learning by revising information on her error and practicing. Though she is not fully able to overcome her mistake, the tool supports S1 to think about her work on a metacognitive level and adopt responsibility for her learning. Thus, S1 uses four FA strategies, while the tool's functionality can be labelled as displaying information or, in case of the *Test* task, providing an interactive environment. Her formative assessment process can be characterised using the FaSMEd framework as shown in Figure 7.

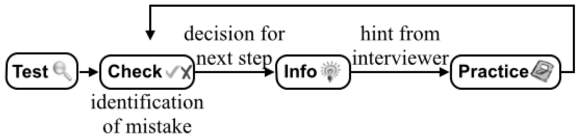

Figure 6: Reconstruction of S1's FA process

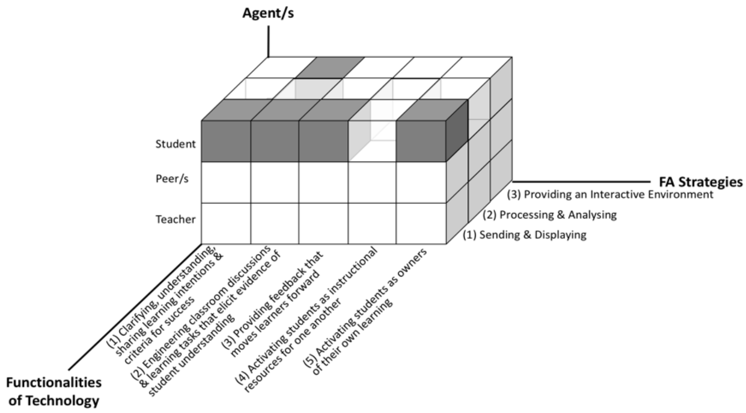

Figure 7: Characterisation of S1's FA process

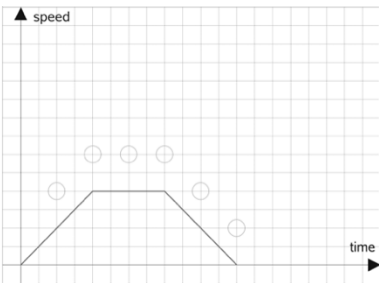

Figure 8: S2's *Test* solution

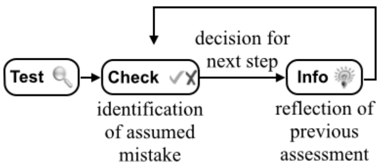

Figure 9: Reconstruction of S2's FA process

S2 also sketched a graph (Figure 8) to solve the *Test* and elicit evidence of her understanding (strategy 2) using the tool's interactive graphing window (functionality 3). In the *Check*, she didn't mark off the statement concerning time being the independent and speed being the dependent quantity. Thus, S2 identifies a supposedly error based on the displayed *Check* statement (functionality 1). Even though she labelled the axes correctly, S2 decided to read the *Info* concerning her alleged mistake and is, thus, adopting responsibility for her learning (strategy 5). When reading the *Info*, she realized: "Oh, that is correct as well because I did it in the same way." She not only states a self-feedback (strategy 3), but also compares the displayed information (functionality 1) to her own *Test* answer and reflects on her assessment (strategy 5). Then S2 went back to the *Check* and marked off the statement correcting the error in her previous assessment autonomously. In conclusion she identifies a correct aspect about her work, which means she now understands a criterion for success (strategy 1).

In summary, S2's work with the digital self-assessment tool can be illustrated as in Figure 9. She works on a diagnostic task, identifies an assumed mistake and decides to gather more information on it. Then S2 identifies an error in her previous self-assessment by comparing her solution of the *Test* to the displayed *Info*. Finally, she corrects her assessment. The analysis shows that within this process,

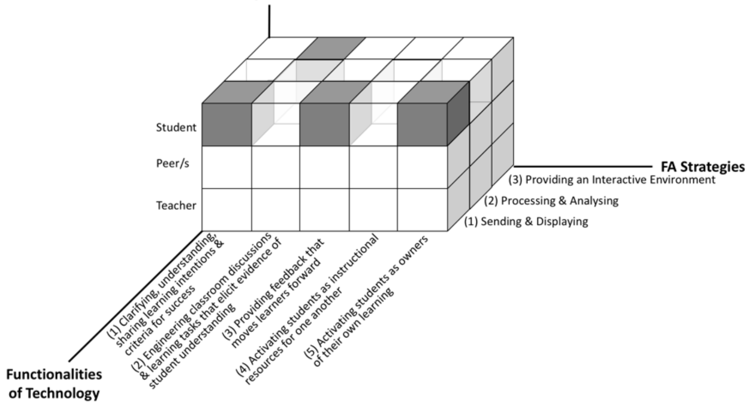

Figure 10: Characterisation of S2's FA process

she uses four different formative assessment strategies, while the tool functions mainly as a display of information and for the *Test* provides an interactive environment (Figure 10).

CONCLUSIONS AND FURTHER STEPS

The analysis of the two cases shows that the tool does have the potential to support students' formative self-assessment concerning their ability to draw a graph based on a given situation. It is the user, who holds the responsibility to identify mistakes and decide on next steps in the learning process. In addition, the tool stimulates students to actively use four different key strategies of formative assessment: the clarification and understanding of criteria for success, eliciting evidence on student understanding, formulating feedback and being activated as the owners of one's own learning.

However, the case studies highlight some constraints of the digital self-assessment tool, which (in the cyclic process of the study) lead to a redesign that is currently being programmed. In the interviews, it became clear that students are uncertain about assessing themselves as they mentioned that they expect validation from either the teacher or the technology. This is why, the redesign focuses on improving students' comprehension of the learning goal, namely the change of representation from situation to graph, and simplifying the learners' self-evaluation. Hence, the static picture of the *Test's* sample solution will be replaced with a simulation of the described bike ride connected to the sample graph as well as the student's own solution (Figure 11). Furthermore, all *Practices* will allow simultaneous views of the student's answer next to a sample solution for easier comparison. The students' interview statements and S1's case, in which she was unable to fully overcome her mistake, revealed that it will not be possible for all students working with the tool to (re)learn the change of representation from situation to graph on their own. Further interventions not included in the tool might be necessary. Therefore, the newest version will save the individual student's work and include a teacher functionality to review students' solutions and enable more effective planning of post-assessment classroom interventions by addressing students' needs more directly.

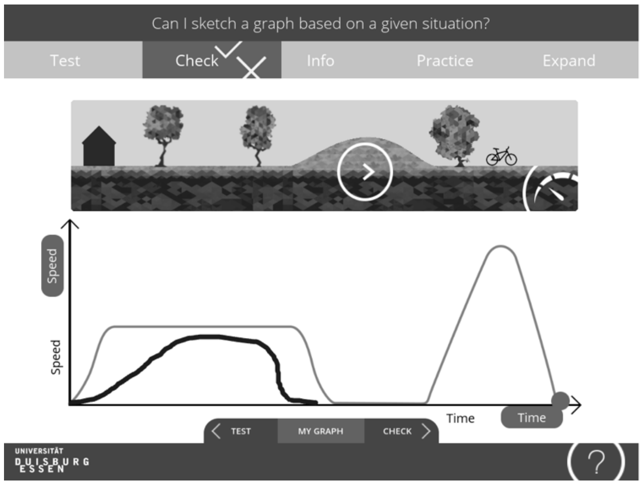

Figure 11: Simulation of bike ride as sample solution of the *Test* task in the tool's current redesign

Furthermore, the two cases show that the tool's functionality can mainly be described as displaying information. To increase the interaction between students and tool, the redesign will include dynamic visualisation for most of the *Info* units. These will enable students to click on highlighted segments of a displayed graph to open and read an explanation. In addition, simulations as described for the *Test's* sample solution, that allow to make connections between the real-life situation and the graph of a function, will be used in some of the *Practice* tasks as well.

Finally, the interviews show that more detailed analyses are necessary to gain a deeper understanding of the students' formative self-assessment processes. While working with the digital tool did not help learners to overcome all of their mistakes, it encouraged them to reflect on their own solutions on a metacognitive level. This seems to be the key for students' success in doing self-assessment. Therefore, a category system for a qualitative content analysis of the interviews is currently being developed. It focuses on three main categories regarding the students': metacognitive activities, tool activities and content-related activities. The aim is to observe which metacognitive activities are prompted through which design aspects of the digital self-assessment tool and how this can help the students' conceptual understanding of the content of functions.

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