DIGITAL MATHEMATICS TEXTBOOKS: ANALYZING STRUCTURE AND STUDENT USES

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The use of digital tools in the Mathematics classroom is an important focus of research in the field of mathematics teaching. In the context of digital textbooks, however, there is still a high need for research and development. Although there are first versions of digital textbooks in the German language, it is unclear what structure and elements digital Mathematics textbooks generally offer or how learners work with them. This contribution addresses this research topic.

Keywords: digital mathematics textbooks, student uses, structure of digital textbooks

THEORETICAL BACKGROUND

When working with (traditional) textbooks, students use the textbook to engage in the field of the book's subject, e.g. mathematics. By doing that, the use of the textbook is affected by its content and structure. Both aspects define the artefact *textbook* and, therefore, influence student uses of the textbook as Chazan and Yerushalmy (2014) pointed out:

> [T]extbooks give teachers guidance on both what and how students should learn. On the one hand, especially initially, textbooks organized the content of what students were to learn and indicated what students needed to know at what age, grade level, or institutional track within schooling. On the other hand, by presenting instructional tasks, textbooks attempt to organize the knowledge that they present in ways that will help make this content learnable. (Chazan & Yerushalmy, 2014, S. 67)

When talking about student uses of textbooks and to see how students work with textbooks in order to engage in the field of mathematics, Rabardel's theory of *instrumental genesis* (Béguin & Rabardel, 2000) is helpful. According to Rabardel, the user turns the artefact textbook into an instrument for learning mathematics during the two intertwined processes (a) *instrumentalization* and (b) *instrumentation*. In the course of *instrumentalization*, the user individually attributes functions to (parts of) the artefact that can be fulfilled, while the *instrumentation* process is concerned with how students select relevant content within the textbook. Rezat (2011) applied this framework to students' paper textbook uses focusing on elements students select for certain learning activities and thus describing student uses of elements within the textbook. He pointed out that students mostly engage in the activities of *practicing* and *solving tasks and problems* and that they usually work with *exercises* and *boxes containing basic knowledge* (cf. Rezat, 2011, p. 171).

In the context of digital mathematics textbooks, the question arises whether the same learning activities and elements can be identified or whether these categories need to be extended.

RESEARCH QUESTIONS AND METHODOLOGY

Transferring and adapting research findings to the context of digital mathematics textbooks involves knowledge on a) the structure and elements of a range of existing digital textbooks concepts and b) what roles/functions students ascribe to certain elements of the digital textbook during a variety of learning activities. These two research perspectives can be framed into the following research questions a) What kind of structural characteristics and elements can be identified in digital mathematics textbooks? and b) What kind of structural elements do students *instrumentalize* when working with digital mathematics textbooks?

Analysing digital mathematics textbooks within the frame of Mayring's *Qualitative Content Analysis* (2008) offers a valid category-developing method in order to identify relevant structural elements. This serves as a basis for the second focus – analysing student uses of the textbook within the frame of *Instrumental Genesis (Rabardel, 2002)*.

Approaching the first research question involves analysing several digital mathematics textbooks (normative analysis). Concerning the second research question (empirical analysis), student uses of the analysed structural elements were videotaped, transcribed and studied based on two levels – *concept-related* and *structural-element-related*.

RESULTS AND DISCUSSION

The main outcome in terms of the normative analysis is that several structural elements could be identified that had not been identified for traditional textbooks (cf. Rezat 2009). This means that a broader variety of types of exercises in terms of their static or dynamic mode were characterised as well as different kinds of feedback modes on how well (or not) exercises were completed. More precisely, we could identify dynamic exercises with a *drag-and-drop* mode where students can select and move small elements within the exercise, *notification exercises* where the user can write down notes, calculation methods or ideas and download the notes in the end, *calculation exercises* where the solution (of that exercise) can be entered in a predefined field and checked for its correctness, or *interactive exercises* which allow students to access mathematical ideas in a dynamic and visualised way. Furthermore, the digital nature of textbooks allows a variety of different feedback modes, i.e. *solution*, *solution process*, and *check solution*. While the first feedback mode gives the learner the correct answer to the task and the second one reveals the solution process and the necessary calculations, only the structural element *check solution* allows a dynamic feedback on the student solution displaying whether the entered solution is right or wrong. The following table lists all structural elements that could be identified for digital textbooks:

Structural Element	**Information**
Additional information	Tip for specific task/exercise
Animation	Visualisation of mathematic content, not modifiable through user
Box with basic knowledge	Formula, definition
Box with hints	General information on current topic
Check solution	Dynamic feedback on the student solution displaying whether the entered solution is right or wrong
Drag-and-drop exercise	Dynamically select and move small elements within the exercise
Dynamic calculation exercise	Solution (of that exercise) can be entered in a predefined field and checked for its correctness
Exercise	Static exercise
Interactive exercises	Access mathematical ideas in a dynamic and visualised way
Notification exercise	Notes, calculation methods or ideas can be entered and

	downloaded
Picture	Static
Solution	Correct answer to the task
Solution process	Solution process and the necessary calculations
Table	Static
Text	Continuous text

Table 1: Overview of structural elements identified for digital mathematics textbooks

Based on the normative analysis or rather the identified structural elements, student uses of the analysed structural elements were videotaped, transcribed and studied. In order to see how new structural elements in digital textbooks effect student uses, the empirical analysis concentrated on how different kinds of feedback options influence student uses of these structural elements. The main outcome of this focus was that individual uses of structural elements on both the *concept-related* level as well as on the *structural-element-related* level could be identified. For example, students referred to their mathematical concept images (in German: "*Grundvorstellungen*") when comparing their results (*concept-related*) <u>before</u> using the structural elements *solution*, *solution process*, or *check solution* as the following example shows:

Student 1 Your book … is definitely smaller than your room.

Student 3 (…) I compared the book with ehh with a small coin.

We can see that although the students compared the book referring to different objects they applied the same underlying concept image, i.e. the concept of 'comparing' (in German: "Vergleichsaspekt") (Weigand et al. 2014, S. 160). Furthermore, student 3 demonstrates the concept image of 'filling out' (in German: "Ausfüllaspekt") (Weigand et al. 2014, S. 161) as the following transcript extract shows:

Student 3 (…) Let's say the book is as big as this [takes a sheet of paper in A4]. Maybe a little bit smaller. Eh … let's say (…) it is up to here [draws a line on the sheet] (…) If I put coins down, then everything will be full of coins. For me, the coin is small. The pinhead is very small. (…) And if you take that to the book, for me, it results in medium-sized.

Student 3 does not only compare the coin to the pinhead and the book, but also fills out the book with a lot of coins whereby the argument refers to the concept image of 'filling out'.

<u>After</u> the learners have used the technological check and correct or wrong assignments are displayed, the statements from the users show reactions on a *structural-element-related level*. For example, students rejected the textbook solution when they did not understand the textbook solution. Another reaction can be seen in the following example as the student is comparing the textbook solution to her own solution:

Student 3 [M]y reasoning was better [than your reasoning].

Here, the student extrapolates from the textbook's feedback to her reasoning and to that of her classmates. Therefore, her argumentation refers to the displayed solutions – hence, to the textbook. A third reaction which could be observed was that of student 2 as he – on the basis of the textbook's feedback – is trying to make sense of the computer solution:

Student 2	[points on the screen] Yes, but … the cent coin is small. Then … the stamp is bigger … then, the stamp must be (…) *smaller than medium* but that is the size of the book so that does not make sense.

All in all, these reactions can be differentiated into the four categories. For a more detailed analysis, see Pohl & Schacht (in press).

Structural element-related Categories
Rejecting the computer solution
Comparison of right or rather wrong solutions
Reconstructing the computer solution based on the computer solution
Reconstructing the own solution based on the computer solution

Table 2: Overview on the effect of the structural elements providing feedback on student solutions on structural-element-related level (cf. Pohl & Schacht in press)

Overall, this contribution shows, besides the student argumentation on a *concept-related level*, the multiplicity of user reactions on the *structural-element-related level* based on the use of individual structural elements with a verifying function. Further empirical analyses of student uses of digital mathematics textbooks should therefore be examined to see whether *concept-related* and *structural-element-related* arguments can collaborate. The two categories "reconstructing the computer solution based on the computer solution" or "reconstructing the own solution based on the computer solution" already suggest a collaboration on both argumentation levels, since the process of reasoning based on the concept images with reference to the structural element becomes apparent.

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