ANALYZING THE TEACHER'S KNOWLEDGE FOR TEACHING MATHEMATICS WITH TECHNOLOGY

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The teacher's knowledge has long been viewed as a strong influence on the students' learning. Several authors have sought to develop procedures to assess this knowledge, but this has proved to be a complex task. In this paper I present an outline of a conceptualization to analyze the teacher's knowledge, based on the model of the Knowledge for Teaching Mathematics with Technology (KTMT) and a set of tasks. These tasks are chosen by the teacher taking into account the potential of the tasks to take advantage of the technology's potential. The analysis of the teacher's KTMT is based on the characteristics of the tasks chosen by the teacher; the balance established between the representations provided by the technology that the tasks advocate; the way how the tasks pay attention to the new issue of seeking for a suitable viewing window; and also the way how the tasks take into account the expectable difficulties of the students in the process of looking for the window.

Keywords: Professional knowledge; KTMT; technology.

INTRODUCTION

The teacher's knowledge is considered as an important requirement for high-quality teaching (Fauskanger, 2015). And many are the authors who have been dedicated to developing characterizations of this knowledge, identifying important aspects of the knowledge required to teach and developing models that articulate specific knowledge in a global and comprehensive structure. And closely related to this intention to characterize the teacher's knowledge, is the desire to assess the knowledge effectively held by the teacher. And this is an issue that has proved to be complex. Several authors (such as Fauskanger (2015) and Schmidt et al. (2009)) point out weaknesses and even question the reliability of the results achieved through the application of some of the instruments developed. There are also several authors who criticize the options taken to assess the teacher's knowledge (such as Rocha (2010) and Schmidt et al. (2009)). According to them, these options are too demanding, in terms of the time required to implement and in terms of the resources required to achieve them.

This article intends to present a theoretical conceptualization to analyze the knowledge of the teacher in a context of technology use. It is a work still in progress, based on the model of Knowledge for Teaching Mathematics with Technology (KTMT) and that is based on the analysis of the tasks proposed by the teacher to the students. At this stage of the work it is only considered the teaching of Functions with the graphing calculator.

The structure of the paper includes a section devoted to a summary presentation of KTMT, followed by a brief critical analysis of the main options taken by two authors who developed tools or strategies to assess the teacher's knowledge. It is then presented the conceptualization that is the object of this article and justified the options assumed. Finally, to clarify the ideas presented, an hypothetical example of application of this conceptualization is discussed. The data presented in this last part are real and have been collected in the course of another study. This means that the tasks were actually implemented by one teacher. However, this is not a real example of application of this conceptualization, once the tasks were selected by the researcher and not by the teacher.

KNOWLEDGE FOR TEACHING MATHEMATICS WITH TECHNOLOGY – KTMT

A look at the knowledge models developed so far, such as Mathematical Knowledge for Teaching (MKT) by Hill et al. (2007) or the TPACK from Mishra and Koehler (2006), suggests the knowledge of Mathematics, Teaching-Learning, Technology, and Curriculum as important domains. These are the basic domains of KTMT, being the Curriculum viewed in a transversal way, influential over all the others domains.

Besides those, KTMT particularly values two sets of inter-domain knowledge developed at the confluence of more than one domain: Mathematics and Technology Knowledge (MTK) and Teaching-Learning and Technology Knowledge (TLTK). This is new knowledge that goes beyond the intersection between knowledge of the base domains. The MTK focuses on the knowledge of how technology influences Mathematics, enhancing or constraining certain aspects. The TLTK focuses on how technology interferes with the teaching-learning process, enhancing or constraining certain approaches.

One of the main intentions behind the design of KTMT, which distinguishes it from other existing models, is to integrate into a single model the research on professional knowledge and on the integration of technology into professional practice. That is why MTK necessarily includes:

- Knowledge of technology's mathematics fidelity, i.e., knowledge of the level of agreement between the results of the Mathematics and the results of the mathematics as presented by the technology;
- Knowledge of the new emphasis that technology puts on the mathematical content (e.g., more intuitive approaches encouraging or requiring a different domain of the influence of the values represented in the coordinate axes on the shape of the displayed graph);
- Knowledge of new sequences of content;
- Representational fluency, involving knowledge of different representations, of how to relate them and how to alternate between different representations and between different forms of the same representation.

And TLTK necessarily includes:

- Knowledge of new issues that technology requires students to deal, including the difficulties they face when using technology and that arise from such use;
- Knowledge of mathematical concordance of the proposed tasks, i.e., the alignment between the mathematics the teacher intended the students to work on and the mathematics the students actually worked;
- Knowledge of the potential of technology for the teaching and learning of mathematics, including knowledge of different types of work and teacher roles that technology becomes possible, knowledge of ways of articulating them and knowledge of the contribution they can bring to mathematics learning.

Finally, KTMT includes Integrated Knowledge (IK). A knowledge held by the teacher that simultaneously articulates knowledge in the base domains and in the two sets of inter-domain knowledge. This is a knowledge developed from the interaction between all the domains and that is characterized by its comprehensive and global nature and at the same time by its particularity, in the sense that it is this knowledge that allows the teacher to maximize the specific potentialities of the technology to provide a better mathematical learning to the students. It is this knowledge that is the true essence of KTMT.

Figure 1 presents a schematic representation of the KTMT model, trying to highlight its pyramidal structure. The colors intend to illustrate the process of development of new knowledge at a higher level. The intention is to represent, for example, the MKT as a new knowledge developing from the knowledge on Mathematics and on Technology. MKT is a new knowledge and not just the intersection of the base knowledge on Mathematics and Technology, in the same sense that orange is a new color developing from red and yellow.

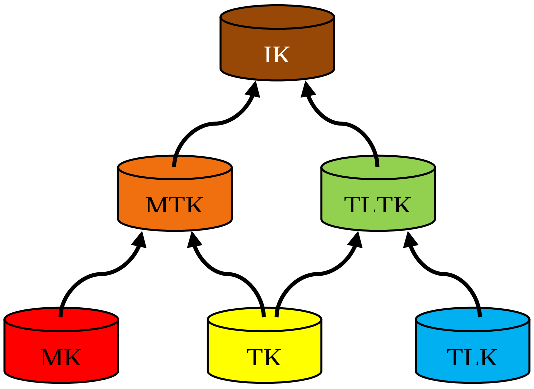


Figure 1. Schematic representation of KTMT model.

ANALYSIS OF THE PROFESSIONAL KNOWLEDGE OF THE TEACHER

The analysis of the professional knowledge held by the teacher has been a concern for several authors, but it has proved to be a complex task.

Angeli and Valanides (2009) assess the TPACK held by the teacher focusing on the task developed by him and a set of five criteria: (1) identification of topics to be taught with technology where the additional value brought by it is patent; (2) identification of representations to transform the content to be taught so that it becomes understandable to the students and in cases where it would be difficult to do so on the basis of traditional methods; (3) identification of teaching strategies that would be difficult or impossible to implement based on traditional means; (4) selection of appropriate technology; (5) identification of appropriate strategies for the introduction of technology in the classroom. The assessment process they use involves expert assessment, peer assessment, and self-assessment, which turns its implementation complex in terms of the structure it requires.

Niess et al. (2009) propose a model of professional development based on a characterization of the use and concerns of the teacher in relation to technology. The authors present four themes (curriculum and assessment, learning, teaching and access), five stages (recognizing, accepting, adapting, exploring and advancing) and a set of descriptors and examples. They do not, however, clarify the reasons behind the choice of these themes to conceptualize the development of knowledge based on a model that is organized in very different domains (knowledge of content, pedagogy, technology and intersections between them). Moreover, as they themselves recognize, the model does not allow a global analysis of professional knowledge, being possible to find teachers at different levels depending on the theme considered.

ANALYZING KTMT BASED ON THE TASKS PROPOSED BY THE TEACHER

The implications of technology's use on mathematics teaching and learning are well recognized (Graham et al., 2003). And Dunham (2000) points to the differences on the tasks proposed and on the students' work resulting from those proposals as the main impact that the graphing calculator can have on teaching. Therefore, it seems pertinent to consider the tasks proposed by the teacher as the basis for the analysis of his KTMT.

According to Goos and Geiger (2000), teaching approaches that emphasize problem solving and exploration, find in this technology a natural and mathematically powerful partner. Increasing the work around open questions and the exploration of concepts by the students is one of the possible consequences of technology integration (Cavanagh, 2006; Graham et al., 2003). The graphing calculator also allows the students to work on data collected by others as well as on data collected by the students themselves (White, 2009). In this sense, the tasks have the potential to elucidate about how the teacher takes advantage of the potential of technology for the teaching and learning of Mathematics.

Ponte (2005) classifies the tasks based on the level of demand that the task places on the students and on the level of structure, taking into account the context of the task (strictly mathematical or from reality). The author then distinguishes the tasks in exercises, problems, explorations or investigations, where the explorations correspond to investigations with a lower degree of difficulty and the modeling tasks are regarded as problems or investigations, depending on the level of structure.

Laborde (2001) considers the tasks in a context of technology use and classifies them into: (1) tasks that are facilitated by the technology, but are not modified by it; (2) tasks where technology facilitates exploration and analysis; (3) tasks that can be done with paper and pencil, but where technology comes to allow new approaches; (4) tasks that cannot be performed without the technology. And the author organizes these types into two groups, depending on how the tasks are facilitated by the technology, but could continue to be implemented without using it; or are modified by it, as in the tasks in which real phenomena are modeled or deductions are made from a set of observations.

Therefore it seems pertinent to start from an analysis of the tasks proposed to access the knowledge on the potential of technology for the teaching and learning of Mathematics held by the teacher and, consequently, his TLTK.

One of the features of the graphing calculator is to allow access to multiple representations (Kaput, 1992), which makes it possible to establish or reinforce connections in a way that would not be possible without the support of technology (Cavanagh & Mitchelmore, 2003), articulating numerical or tabular, symbolic or algebraic and graphical representations (Goos & Benninson, 2008) and fostering the development of a better understanding of Functions (Burril, 2008). As Kaput (1989) points out, the connection between different representations creates a global vision, which is more than the joining of the knowledge relative to each of the representations. And the author emphasizes how the technology allows a full exploration of the numerical and graphic approaches in a way that until then was not possible, thus favoring an integrated approach of the different representations and consequently the development of a deeper understanding. Thus the use of multiple representations has the potential to turn learning in a meaningful and effective experience.

Despite the importance of working with different representations and the teachers' concern about articulating and balancing their use, Molenje and Doerr (2006) found that the use of algebraic and graphic representations are dominant with respect to numerical representation. Moreover, when teachers actually use all the three representations, there tends to be a pattern in the way they do it. This pattern in the alternation between representations tends to be copied by the students, and this makes it difficult for students to develop the desired fluency between the different representations.

In this sense, an analysis of how the tasks proposed require the use of different representations and, when they do, an analysis of the characteristics that can be identified in their use, is an indicator of the representational fluency of the teacher and, consequently, of his MTK.

The integration of the graphing calculator in the teaching and learning of Mathematics forces the students to take mathematical decisions that until then they never had to face (Cavanagh & Mitchelmore, 2003). As the authors point out, the graphing calculator requires the students to make an adequate choice of the scale and of the values of the viewing window, to know how to deal with situations in which no graph is observed, or in which only a partial view appears. This is mathematical content that the students were not used to deal with and, in this sense, the way how the teacher includes it in the tasks proposed to the students may be another indicator of his MTK.

According to Cavanagh and Mitchelmore (2003), the main difficulty faced by the students when the technology becomes available in the classroom concerns to the process of finding a suitable viewing window. And the authors attribute the origin of this difficulty to the previous mathematical experience of the students, where all the graphs were drawn with paper and pencil using a referential where the values represented were almost always the same. This highlights the attention that the teacher needs to give to these issues, reflecting on the new emphases that the use of technology tends to place on mathematical contents. It is therefore crucial that the teacher manages appropriately the form and, above all, the moment when the students face these difficulties. Actually, according to Cavanagh (2006), it is fundamental that the teacher promotes a gradual contact with potentially problematic situations, ensuring that the students do not face them too soon. And an analysis of the tasks proposed will undoubtedly allow us to understand how the teacher considers this question. It will thus be another way of accessing elements of his TLTK.

We have thus identified a set of aspects that allow us to access elements of teacher's KTMT and, consequently, to assess the teacher's professional knowledge from an analysis of the tasks proposed by the teacher (see figure 2 for a synthesis).

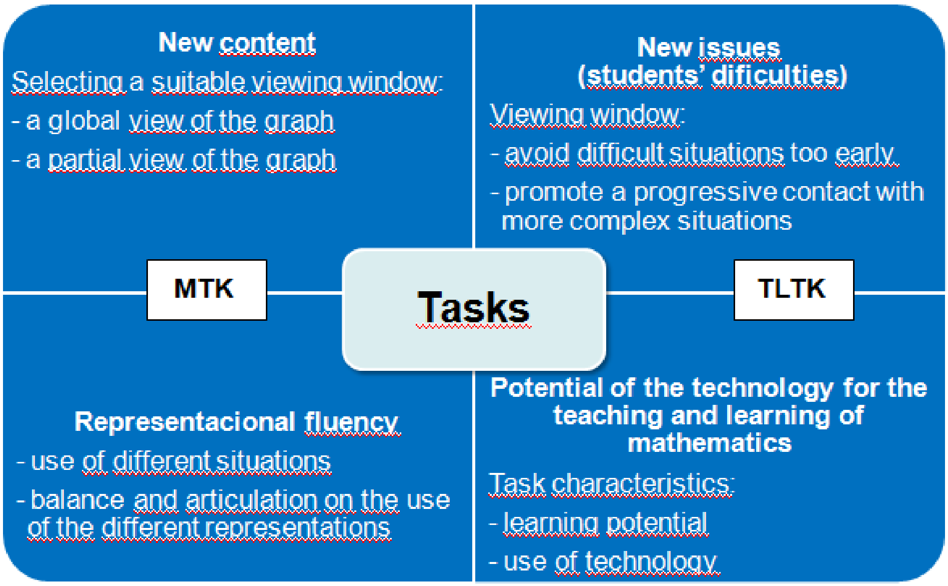


Figure 2. Synthesis of aspects to analyze in the tasks to access KTMT.

EXAMPLE

Let us suppose that a teacher selects four tasks to illustrate the work done by his students of a 10th grade class when they study functions using the graphing calculator.

The first of the tasks selected was implemented in the third lesson (L3) and asked the students to study some families of quadratic functions: ax^2, ax^2+c, $a(x-h)^2$, $a(x-h)^2+k$ ($a\neq0$). Students should observe graphs of their choice of functions in each of the families under study and conjecture about the effect on the graph of changing the parameters. Then they are asked to report their conclusions.

In the fourth lesson (L4), the task consists on a situation with real context, where the number of bacteria, in thousands, of a certain colony, is given by $N(h)=-h^2 + 4h + 9$, where h represents the elapsed time. Questions relating to the number of bacteria at certain times and to the period of time when the number of bacteria is higher or lower than certain values are then placed. The students are expected to analyze the function and calculate function values, find the object of a certain image and calculate the zeros of the function.

In the task proposed on the tenth lesson (L10), the students were given a graph on paper (see figure 3) and informed that the function represented is a 3rd degree polynomial function with a root for $x=\sqrt{3}$. Then they are asked to draw the referential so that the function is odd. After finding that the expression of the function is $f(x)= 1/6\ x\ (x^2-3)$ (using analytical methods), the students were asked to find the relative maximum and minimum of the function.

Figure 3. Graphical representation of the function on the task of 10th lesson.

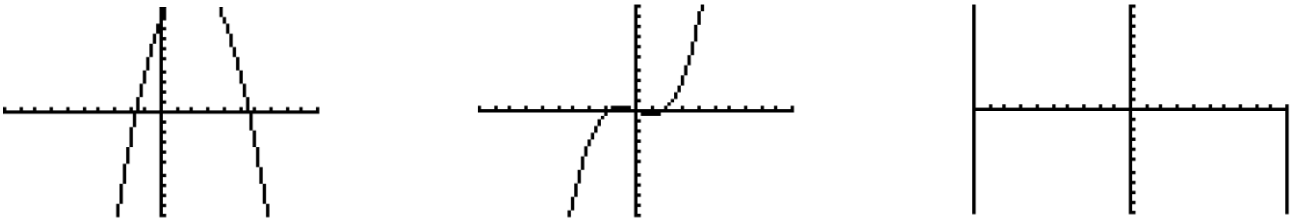
In the task proposed on the twelfth lesson (L12), the students were asked to graphically solve the inequality $x^3 – 100x \leq 10x^2 + 100x$.

The analysis of the tasks supposedly chosen by this teacher shows a reduced diversity in what concerns to the type of tasks. Adopting the classification developed by Ponte (2005), the task proposed at the 3rd lesson (L3) can be classified as an exploration. The other three tasks are exercises where the students rely on already known strategies. In all the cases the students used some procedure to get the intended answer from the calculator (find a zero or a maximum, calculate a value of the function or the object of a certain image, find the intersection points of two functions).

In what concerns the relevance of the technology to solve the tasks, in most of these tasks the technology does not assume a central role. Dunham (2000) points to the potential of technology to allow the exploitation of different kinds of tasks, as the main impact of technology integration. But in this set of tasks, sometimes the use of technology is not even required to solve the task. That is the case of tasks on the 4th and 12th lessons (L4 and L12). However, solving the tasks with and without the technology is slightly different. According to the classification of Laborde (2001) for tasks in a context of technology integration, those are tasks that can be done with paper and pencil, but where the technology comes to allow new approaches. The task proposed on the 3rd lesson (L3) is a task that cannot be done without the technology. Actually, it is the technology that allows the students to explore the different graphs, trying to find the impact of changing the parameters on the graph of the function. The task solved on 10th lesson (L10) requires the use of technology to calculate the maximum and minimum values of the function because the students have not learned the analytical way of doing it.

Therefore it seems that the teacher's choice of tasks does not take into account the different types of tasks available, neither the potential of the technology to change the characteristics of the students work and to promote students' mathematical understanding.

Concerning the way in which the tasks require the students to deal with the viewing window, it is possible to identify mainly two situations. The task proposed on the 3rd lesson (L3) leaves to the students the decision about the functions to represent graphically. However, if they do not choose big values for the parameters, it will be possible to represent the functions using the standard window of the calculator. On the 4th lesson (L4), the task includes a function whose graphic is not completely visible on the standard window (see figure 4a), nevertheless it is possible to understand the behavior of the function from that view of the graph and to solve the task without changing the viewing window. On the task from lesson 10 (L10), the graph of the function in the standard window is somehow compressed around the x-axis (see figure 4b), but once again it is possible to intuit the behavior of the function and to solve the task without any change on the window. So this three tasks can be solved using the standard window and do not require knowledge about a suitable choice of the viewing window. The task proposed on the 12th lesson (L12) is different. In this case the standard window shows two lines that do not allow an understanding of the two functions represented (see figure 4c). Although the image immediately suggests that this is not a good representation of the graph, finding a suitable window is not trivial.



(a) (b) (c)

Figure 4. Graphic representation of the functions from 3rd, 4th and 10th lessons on the standard window.

The approach to the viewing window, suggested by these tasks, indicates that the teacher is aware that finding a suitable viewing window is a complex task for the students. As a consequence the teacher seems to avoid complex situations too early (as recommended by Cavanagh (2006)). Nevertheless, there is no evidence about the teacher's knowledge on the different types of situations related to finding a suitable viewing window. In fact, the tasks do not include situations of hidden behavior, incomplete view or partial view, suggesting that the teacher could have moved from situations where the standard window allows the students to solve the task (such as the tasks in L3, L4 and L10), to situations of simultaneous incomplete and partial view (the task in L12). There is also no evidence about the teacher's knowledge in what concerns the different mathematical knowledge needed to deal with each of the different types of situations.

In what concerns the work around the representations, it is again possible to identify two situations related to how the different representations are articulated. On the task proposed on the 3rd lesson (L3), the students start from an algebraic representation and move to a graphic representation, starting a cycle of alternation between these two representations, trying to make sense of the impact of the parameter on the graph. On the other three tasks proposed the students start from an algebraic representation, move to a graph representation and then to a numerical representation (on the graph). This option suggests a preference for a pattern in the alternation between representations. It is also evident an absence of the use of tabular representation.

Figure 5 presents a schematic synthesis of the knowledge of this teacher.

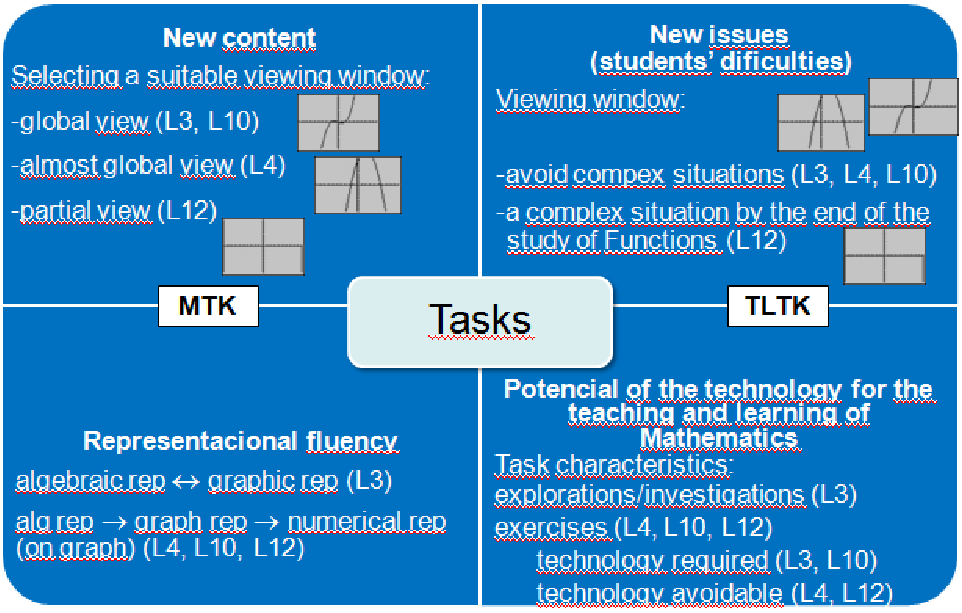


Figure 5. Synthesis of the teacher's KTMT analysis.

This teacher evidences having knowledge about important aspects, however it is possible to identify domains where his KTMT could be developed. That is, domains that should be considered when planning a professional development program for this teacher. That is the case of developing a deeper knowledge on the type of tasks taking advantage of the potentialities of the technology for the learning of mathematics; of the different types of situations requiring a change on the viewing window and of the mathematical knowledge needed on each one; and of the different ways of articulate and balance the representations available on the calculator.

CONCLUSION

In this article we present a proposal for the analysis of the professional knowledge of the teacher based on the KTMT's model and a set of tasks chosen by the teacher as representative of what was done with the students using technology. A reflection based on the fictitious example presented suggests that this proposal has the potential to access aspects of professional knowledge, nevertheless some criticism of the options assumed is inevitable. And the main one is related to the way how this proposal for analyzing the teacher's knowledge ignores all the aspects related to the implementation of the tasks.

In fact, although important, the tasks proposed alone do not characterize the teacher's practice. As Boaler (2003) emphasizes, one must take into account the complexity of practice and keep in mind that the same task can support different practices. In other words, we cannot forget that even the most meritorious task does not necessarily constitute the inevitable source of a productive learning environment, since what might at first be a rich question, that appeals to the students' exploration can, depending on the teacher's implementation, lose its potential and become a trivial exercise.

And if it is undeniable that the absence of elements regarding the implementation of the tasks by the teacher is a reality, it is not so clear that it is effectively a significant limitation. In the worst case (where the implementation of the task in the classroom reduces its potential), we will always have a majorization of the teacher's professional knowledge. And this because if there are references to a reduction of the cognitive demand of the task during its implementation, the opposite does not seem to be usual. And this majorization of the teacher's knowledge will continue to be useful and relevant information.

This proposal intends to assume a simple structure, still allowing access to relevant information. It aims to avoid complex analyzes carried out by multiple actors, as in the case of Angeli and Valanides (2009); or multiple interpretations of the teacher's knowledge according to the domain considered, as in the case of Niess et al. (2009).

Moreover, this is not the only proposal which does not include aspects relating to the teacher's practice. The work of Hill et al. (2007), which is based on a closed-ended questionnaire to which teachers respond, is well known. The number of authors who developed their research based on the ideas of Hill et al. (2007) suggests that it is possible to develop useful strategies to analyze the teacher's knowledge that do not include elements from the classroom practice.

This is, however, a proposal that is still in an early stage of its development. It is now important to refine it and to analyze in more depth the contributions it can bring, in particular by considering its integration within existing ones.

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