TEACHING WITH GEOGEBRA: RESOURCE SYSTEMS OF MATHEMATICS TEACHERS

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This paper reports from a study of use of the dynamic mathematics software, GeoGebra. This study used Ruthven's (2009) Structuring Features of Classroom Practice model to analyse the classroom practices of three teachers in English secondary schools. Here the particular focus is on how teachers manage their resource system with the use of GeoGebra. The main conclusion was that the stage teachers were at in terms of learning to teach with this software indicated differences in regard to establishing a functioning resource system especially in teaching the operation of the software, preparing the dynamic files and the choice of tasks. It became evident that the most experienced teacher was more fluent in managing his system of resources.

Keywords: classroom practice, mathematics teaching, resource system, dynamic mathematics software

INTRODUCTION

Although the influence of new digital technologies has been increasing over recent decades, their incorporation into mathematics education has been slow. It has become apparent that teachers have a central role in the integration of technology in mathematics classrooms, and that this issue needs more attention by research. Understanding the process that teachers go through to appropriate technology effectively into their instruction is of crucial importance to help facilitate successful integration. In this report the concept of 'resource system' will be used to describe a central aspect of the instructional environment of classrooms bearing on the use of technology. Ruthven (2009) has drawn attention to this concept as one of five key structuring features of classroom practice (alongside working environment, resource system, curriculum script and time economy), which shape the use of technology in lessons and the kinds of professional knowledge required. Resource system relates to collection of didactical tools and materials in use, and coordination of use towards subject activity and curricular goals (Ruthven, 2014).

With the influx of new technologies in schools, on the one hand classrooms are filled with resources, which can be considered as providing more opportunities for learning. On the other hand, the usability and adaptability of new technologies can present challenges for some teachers and students since "resources are not self explanatory objects with mathematics shining clearly through them" (Adler, 2000, p. 207). Researchers have shown that the challenge of adaptability of dynamic geometry software (DGS) lead the majority of teachers to use them in the more conventional types as "a reduced and static use of the possibilities of the software", and "the absence of autonomous experimentation by students" (Laborde, 2001, p. 299). Additionally, Ruthven et al. (2008) argues that there is an 'interpretative flexibility' in the incorporation of dynamic geometry software into educational practices. The researchers highlight the difference between how mathematics educators interpret DGS to be used in classrooms (e.g. more open student exploration) and how more typical teachers use it in their teaching.

Where dynamic geometry has entered mainstream classrooms, it appears to be used to support more established forms of pedagogical practice, notably student activity directed towards empirical confirmation of standard curricular results, often through guided discovery, as already prevalent in the teaching of geometry in many educational systems. (Ruthven et al., 2008, p. 314)

In terms of the design of tasks, Laborde's (2001) study indicated that it was challenging for teachers to go beyond textbook tasks when using DGS. Development of task design scaled from almost traditional geometry tasks, to tasks that could only be approached in a dynamic geometry environment. The former indicated tasks typically applied in a paper-and-pencil environment often supported by a textbook and by applying available tools such as compasses and ruler. Using DGS on such tasks only facilitated drawing the shapes more accurately and quickly. In this regard, Laborde emphasised the fact that integration of DGS into mathematics teaching is a lengthy process. On the other hand, Monaghan's (2004) study indicated that most of the teachers managed to leave aside their textbooks in their technology-based lessons. However, the tasks in the worksheets "emphasised students' management of the computer software per se" as mentioned before. Similarly, findings from Erfjord's (2011) study investigating teachers' initial use of DGS in Norway indicated that two teachers used DGS with prepared instructional material focusing on the technical aspects of using DGS and had little explicit focus on the mathematics by mostly demoing for students what they needed to do in technical terms. This offered students a good background to further develop their ability to use compasses. Dragging mode was utilised for checking the accurate use of DGS rather than for provoking mathematical interpretation with the aim of improving students' abilities of using compasses. Furthermore, Assude (2005) also reported that in terms of use of compasses and Cabri there was not "any major changes with regard to the broad types of tasks: construction, description and property identification." (p. 192). Similarly, Ruthven, Hennessy, and Deaney (2005) argue that the typical use of DGS proposed by teachers in England is to let students work with geometrical properties utilising the dynamic dragging-function; further that many teachers in their teaching tried to control and constrain student work in order to avoid students spending too much time on the exploratory affordances in the DGS.

In this respect, this paper examines how three mathematics teachers establish their resource system with the use of GeoGebra –a dynamic mathematics software- in ordinary classrooms. The specific research question is: "What aspects of teachers professional knowledge emerge in relation to working with their resource system?"

RESEARCH CONTEXT

This paper reports on the 'resource system' aspect of the classroom practice of three English secondary-school mathematics teachers, associated with their use of GeoGebra for mathematics teaching. GeoGebra is an open-source educational software package, which provides dynamic mathematical representations (Hohenwarter & Preiner, 2007). We chose to study GeoGebra lessons in particular because of the interest that this software has attracted amongst teachers in England. After teachers agreed to participate in the research, the first author visited their schools to discuss their timetable and to find out when they were planning to make some significant use of this software. Observations and interviews then took place at a time agreed in advance with each teacher at his or her convenience. In this paper we focus on a type of mathematical topic which all three teachers chose to teach with GeoGebra, concerned with geometrical transformations.

The three teachers participating in this research have rather different profiles:

– an experienced teacher (pseudonym Chris) and Advanced Skills Teacher (a recognised grade of classroom teacher within the English school system, also taking special responsibility for leading professional development), who utilises new technologies in a progressive way in mathematics instruction. He taught the topic over a series of four lessons to a high attaining Year 9 class.

– an experienced teacher (pseudonym Susan) in terms of mathematics teaching but a novice technology user who is in her early stages in using GeoGebra. It was the first time she had integrated GeoGebra into her teaching of this topic to the extent of actively involving students in dynamic exploration. She taught the topic over two lessons to a lower set Year 7 class.

– a less experienced teacher (pseudonym Tom) in terms of mathematics teaching (4 years teaching experience) but familiar with the use of GeoGebra in mathematics teaching. He started using GeoGebra regularly in his teaching from the beginning of his career. He taught this topic over a period of three lessons to a Year 10 class (set 2 out of 4).

Classroom Observations: A semi-structured, non-participant observation approach was adopted for which the SFCP framework as an interpretative lens provided guidelines.

Teacher Interviews: Semi-structured post-lesson interviews were conducted in order to clarify the observed lessons and the professional thinking behind them according to the key themes of the SFCP framework.

RESULTS AND DISCUSSION

The resource system for all the teachers consisted of prepared dynamic files involving use of GeoGebra.

Susan employed GeoGebra with a pre-developed (paper) worksheet including four tasks, accompanied by prepared dynamic versions of each task for students to experiment with in GeoGebra. She borrowed these tasks from a Geogebra website. The purpose of these lessons was for students to learn a specific type of transformation, reflection. Susan's use of GeoGebra for this topic aimed at easing students' learning by permitting them to observe and think about the processes involved. In addition, Susan reported that GeoGebra speeded the learning process up by allowing the students to engage with more examples that were unusual to see in the traditional paper-and-pencil environment. She used the school's Virtual Learning Environment (VLE) for students to download/upload the tasks. Use of the VLE also aimed at sharing students' work with the whole class. All the reflection files required, first, students' predictions of where they thought the shapes would end up after reflection, second, students checking their predictions in GeoGebra to see if they were correct. In addition, for some tasks, students were told to move some of the points at the corner of the shapes and observe the changes.

Tom used teacher-created files involving use of GeoGebra for these lessons. Tom considered the dynamic and instant nature of GeoGebra as essential characteristics. With the aid of GeoGebra, students could be more curious and see what happens in a diagram, which Tom believed was distinctive to Dynamic Mathematics Software use in comparison with traditional methods. One of the prepared files was the Reflection file. Students were asked to draw the reflections of a number of shapes by using the polygon tool, and then check their answers by using the reflection tool to see if they were right or wrong.
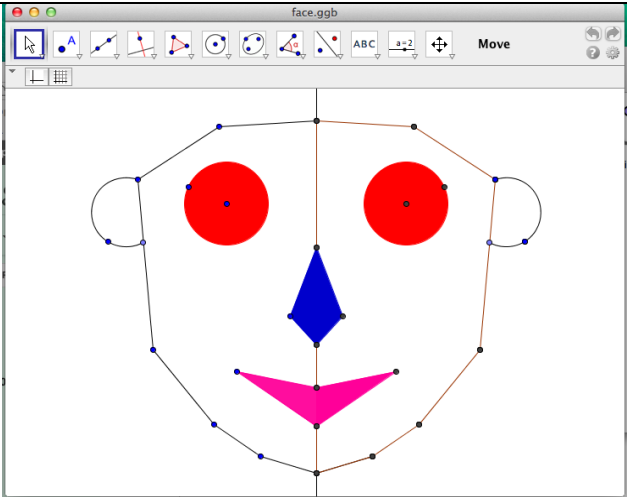
Chris also used teacher-created dynamic files involving use of GeoGebra. As an example, in the second lesson the teacher set up a file, where there was an object and its image locked so that the

students were not able to move anything. The assignment was to locate the line of symmetry. Working with these prepared dynamic files, Chris encouraged his students to explore different paths to reach the same mathematical solution and placed less emphasis on finding the exact answer. In this case, he considered the use of GeoGebra for this topic as a tool to help students' thinking.

Four subthemes emerged, reflecting different aspects of teachers' professional knowledge in relation to working with this resource system.

Operation of the software

Susan's class was novice in GeoGebra use; therefore she allocated some lesson time to showing students the general set up of GeoGebra and the VLE, prior to the Transformations lessons, for students to get familiar with the technology. At the beginning of the first lesson, Susan did a technical demo on the IWB in order to show students how to go onto the VLE and find the GeoGebra files for students to download and work on and then upload back the finished activity to the VLE. In addition, she explained the tasks on the paper worksheet and the dynamic version of those tasks found through the VLE. Then, Susan provided students with a task (that was related only to technical aspects of the software) for them to accustomed to the software (see Table 1).

Table 1. The face activity and instructions for the activity	
	• Reduce the size of his eyes • Make him look sad • Give him two earrings (chose a shape: polygon or circle). • Make one on the left and then reflect it in the line using the reflect tool • Give him eyebrows (use line segment and reflect it in the line)

Tom demonstrated and explained a number of GeoGebra techniques to students when and as necessary during different segments of his lessons. For instance, at the beginning of the first lesson, Tom demonstrated some new skills that are as following: how to copy and paste, which was different in the new GeoGebra version, how to use a vector from a point in GeoGebra, how to set a slider. His class had used GeoGebra before and had the basic knowledge of the software, such as how to find and open GeoGebra and get access to the tool bar.

Chris began the lesson by demonstrating how to operate the software: how to open the software, where to find the prepared files for transformations, where the related icons were located, as well as how to use the dragging tool. Then, he provided students with a dynamic file to explore the software by themselves the students were using GeoGebra for the first time for this particular topic (see Figure 1). With this dynamic file, he required students not only to learn the software but also "to look for interesting things" in the diagram and "explore the mathematics".
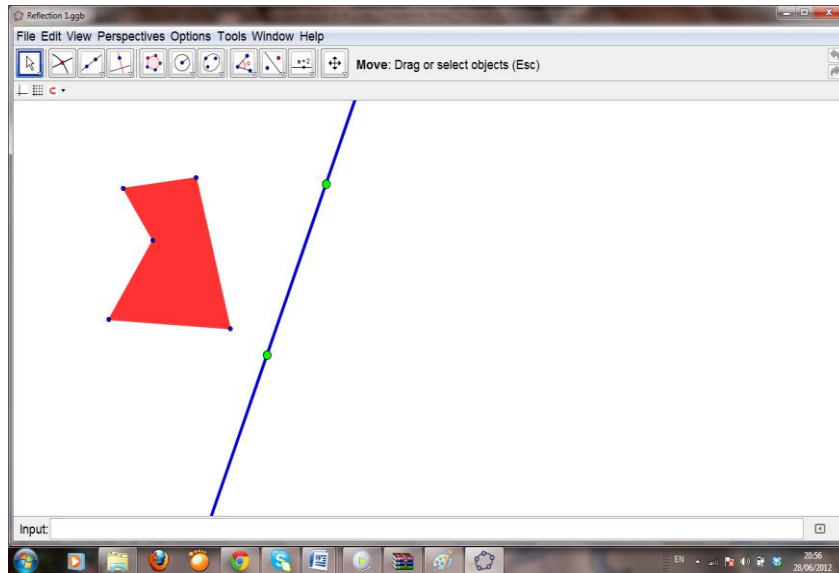
Figure 1. Reflection file that students worked on during Lesson 1

Prepared files involving use of GeoGebra

All teachers provided students with pre-designed dynamic files so that students could work on transformation concepts.

Level of professional knowledge underpinning pre-designed dynamic files was varried:

- Susan borrowed a pre-developed (paper) worksheet and accompanied by dynamic files for students to experiment with in GeoGebra. According to Susan, the use of such worksheet "helps the teacher" by acting as a "crutch" especially for those new to learning to teach with GeoGebra.

- Tom designed his own tasks involving GeoGebra use. On the basis of his experience, Tom found starting from a blank page "too much" for students as "there is quite a lot going on at the background of the program". He prepared dynamic files for students "to get them just see something in action, understand what the idea of translation or rotation is and to be able to then visualise their imagination."

- Chris had well-established teaching repertoires with the use of GeoGebra and planned investigative lessons. He graded students' use of technology by providing prepared files for the first five lessons and then for the last two lessons asking them to start from blank pages to create their own files with the guide of instructions. This implied that Chris saw value in eventually having the students create their own files. As believed by Chris, working with prepared files to start with gave the students more time to focus on the mathematics and their thinking, and thus increased their learning time.

These case studies illustrated a relationship between the technology experience level of teachers and the use of ready-made resources. Susan as a novice technology-using teacher found it more appropriate to start with files and worksheets already prepared by someone else: this gave her more confidence and also alleviated the time pressures she felt. On the other hand, Tom, who has made more use of GeoGebra in teaching than Susan, created the dynamic files himself taking into account of his previous experience. Chris, who is the most experienced of the three in teaching with GeoGebra, also managed students use of GeoGebra gradually by allowing them to gain confidence

in working from scratch with GeoGebra. This development of activities appeared very similar to the progression reported by Laborde (2001). Following four teachers for three years to understand how teachers novice to technology integrated it into their teaching, Laborde found that they gradually granted control of technology and the learning environment to students.

Choice of tasks

Tom and Chris designed the tasks for students themselves whereas Susan borrowed a pre-made worksheet from a GeoGebra website. In this sense, Susan was at the stage where she needed pre-made worksheets/tasks to incorporate GeoGebra into her lessons so that she could follow those worksheets to set a lesson agenda. The tasks designed by Chris were more open ended in the sense that they allowed the students to discover different ways of solving a problem. Furthermore, Chris encouraged the students to take various directions and focused on students' thinking rather than the result itself, which in turn enabled him to create whole class discussion of different ideas and approaches. However, the tasks prepared by Tom were more structured in that there was not room for students to consider from multiple perspectives. In addition, he mostly relied on questions from a textbook which he considered the best he had ever used because it was very close to the current examination specifications. This implied that he wanted his students to practice with questions that could also directly prepare them for examinations.

In this sense, Tom and Susan both tended to use closed-ended tasks with the focus on students getting the questions correct. However. Chris used open ended tasks that allowed students to discover different ways of solving a problem. He had much richer repertoire of interactional moves related to open ended GeoGebra based tasks.

Again, Laborde (2001) emphasised that teachers experienced the design of tasks to be employed with computer software as a challenging process (exemplified in her study by the Dynamic Geometry Software, Cabri). Both Laborde (2001) and Monaghan (2004) found that most teachers used and designed worksheets characterised as having elements of control or guidance, which were usually in accordance with teachers' use of tasks from textbooks in non-technology lessons. Thus, Susan's and Tom's choices to use files/questions intended to closely guide and support their teaching of the lessons and the individual student work in GeoGebra are not surprising. However, Chris, as an experienced teacher and technology user, moved beyond that and came up with his own resources, which were not inspired by paper-and-pencil activities. This suggested that he had come to the point that he no longer found tasks that replicated those from textbooks to be learning productive.

Promoting independent learning through the use of software

The tasks to be utilised with GeoGebra for Transformations lessons attempted to stimulate students' use of a trial and refinement process. All the teachers pointed to the ways in which use of the software could facilitate the process of self-testing and self-correcting. Hence, the main new aspect that these case studies provided evidence for was that activity formats depended more on processes through which the students made a prediction and the technology provided feedback on it rather than (as the teachers pointed out) the teacher or other students validating – or invalidating – it. That shifted the role of the teacher towards becoming an organizer/observer of this process. In addition, technology appeared to be conducive, even essential, to this activity format providing for the necessary levels of interactivity and immediate feedback.

In this connection, Predict-and-test was part of the logic of all the teachers' lesson agendas for handling this topic. In Tom's and Susan's cases, choice of the tasks making use of GeoGebra aimed to enable pupils to check the results of Transformations already done 'by hand'. In this regard, they

had limited professional knowledge regarding interactional moves around tasks. However, as has already been discussed, Chris provided students with more open-ended tasks that allowed students to have different ideas and encouraged them to test their ideas with the dynamic software. The aim was to promote students' skills "in terms of their development as a mathematician". He was already used to discursive open ended approach.

In summary, the three teachers clearly demonstrated how establishing a functioning resource system was an important issue for them and they illustrated different stages in achieving that (see Table 2). Susan was at the stage where consideration of the resource system was the focus as she and the students needed to learn how to use the particular software. On the other hand, Tom was more confident in operating the software and constructed his own files for students to work on. In Chris's case, he was at the stage where a coherent set of resources was already developed as a functioning resource system and knew how to prepare his students in order for them to be able to use those. In this sense, it is possible to see the transition to actually creating geometric constructions and other dynamic figures from scratch as teachers gain more confidence in operating the software. In terms of design of technology-based tasks, these case studies illustrated that the experience level also suggests that there is a change from regular forms of extension activity to things that need high-level thinking. Considering the most experienced teacher in the study, it is clear that for teaching the topic Chris placed far more emphasis on the development of high-level mathematical reasoning/thinking in comparison with Susan and Tom. Furthermore, between Tom and Susan, the evidence highlights that Tom, drawing on his experience, put more thought into designing his task by focusing on students' thinking.

Table 2. Characterisation of Novice/Expert teachers that relates to resource system			
	Susan (limited)	Tom (developing)	Chris (sophisticated)
Resource System	Just embarked on establishing a functioning resource system with GeoGebra for her classes. Prioritized mainly developing instrumental knowledge to make it part of the resource system, first for herself personally and then also for her students. Started with files and worksheets already prepared by someone else: this gave her more confidence and also alleviated the time pressures she felt.	Had confidence in operating the software since he previously made use of GeoGebra in teaching. Created dynamic files himself taking into account of his previous experience so that students could focus on mathematical content rather than dealing with complexity of software.	Had already established a set of resources as a functioning resource system. Managed students use of GeoGebra gradually by allowing them to gain confidence in working from scratch with GeoGebra. Designed more open-ended tasks that allowed students to discover different ways of solving a problem.

REFERENCES

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education*, 3(3), 205–224.

Assude, T. (2005). Time management in the work economy of a class. A case study: Integration of Cabri in primary school mathematics teaching. Educational Studies in Mathematics, 59(2), 183-203.

Drijvers, P. (2012). Teachers transforming resources into orchestrations. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), From Text to 'Lived' Resources (pp. 265–281). New York: Springer.

Drijvers, P., Doorman, M., Boon, P., Reed, H., & Gravemeijer, K. (2010). The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. Educational Studies in Mathematics, 75(2), 213–234.

Erfjord, I. (2011). Teachers' Initial Orchestration of Students' Dynamic Geometry Software Use: Consequences for Students' Opportunities to Learn Mathematics. *Technology, Knowledge and Learning*, 16(1), 35–54.

Laborde, C. (2001). Integration of technology in the design of geometry tasks with Cabri-geometry. *International Journal of Computers for Mathematical Learning*, 6(3), 283–317.

Monaghan, J. (2004). Teachers' activities in technology-based mathematics lessons. International Journal of Computers for Mathematical Learning, 9(3), 327–357.

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. Education & Didactique, 3(1), 131–159.

Ruthven, K. (2014). Frameworks for analysing the expertise that underpins successful integration of digital technologies into everyday teaching practice. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The Mathematics Teacher in the Digital Era* (pp. 373–393). Berlin: Springer.

Ruthven, K., Hennessy, S., & Deaney, R. (2005). Incorporating dynamic geometry systems into secondary mathematics education: Didactical perspectives and practices of teachers. In D. Wright (Ed.) *Moving on with Dynamic Geometry* (pp. 138-158). Derby: Association of Teachers of Mathematics.

Ruthven, K., Hennessy, S., & Deaney, R. (2008). Constructions of dynamic geometry: A study of the interpretative flexibility of educational software in classroom practice. *Computers & Education*, 51(1), 297–317.

Ruthven, K., Deaney, R., & Hennessy, S. (2009). Using graphing software to teach about algebraic forms: A study of technology-supported practice in secondary-school mathematics. Educational Studies in Mathematics, 71(3), 279-297.