SPATIAL–SEMIOTIC ANALYSIS OF AN EIGHTH GRADE STUDENT'S USE OF 3D MODELLING SOFTWARE

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The aim of this paper is to analyse the emergence of spatial–semiotic resources attached to an eighth-grade student's use of 3D modelling software while solving certain spatial tasks. The data comes from a task-based interview and it is analysed within a spatial-semiotic lens, including different kinds of resources not only based on the discourse, but also based on extra-linguistic expressions such as that sketches and gestures. The results of the study show that generally the student's reasoning steps explored a viewpoint for adding or removing cubes by use of the 'orbit' and 'select' tools, using ready-made mental pictures derived from completed steps, linking 2D and 3D representations through spatial visualisation and spatial orientation, emergence of spatial vocabulary including his strategies and generalizations.

Keywords: Spatial thinking, Spatial–Semiotic lens, 3D modelling software, Multimodal paradigm.

INTRODUCTION

The acts of thinking, constructing and expressing meaning through digital technologies are generally beyond words, but they can also be interlaced with our gestures, mimics and sometimes with specific sketches. Consequently, involvement of our sensory-motor functions' productions in our communication can be considered to be a *multimodal* process (Arzarello & Robutti, 2008). Following a multimodal paradigm, to interpret specific *signs* that emerge in communicating and/or expressing meaning, *semiotic perspectives* have received robust attention from mathematics educators (Arzarello, 2008; Godino, Batanero, & Font, 2007; Ng & Sinclair, 2013; Presmeg, Radford, Roth, & Kadunz, 2016).

Spatial thinking is a core concept in the teaching and learning of mathematics, which can be defined as an amalgam of different sub-skills in relation to geometric reasoning. Because of its importance, a number of epistemological analyses were conducted to elaborate and explain how individuals think spatially when they commence a mathematical task, and specific *spatial images* (Presmeg, 1986) and specific *processes* for 3D geometry and visualization (Bishop, 1983; Gutiérrez, 1996; Yakimanskaya, 1991) have been defined by researchers.

In this work, we acknowledge a combination of two paradigms, namely the synergy between the semiotic perspective-multimodal paradigm and spatial thinking, and consider the following research question: what kind of spatial–semiotic resources emerges when an eight-grade student solves spatial tasks with a 3D modelling software?

CONCEPTUAL FRAMEWORK

In order to analyse classroom activities with a *spatial–semiotic* lens (S-SL), Turgut (2017) proposes a conceptual framework based on the hypothesis that thinking spatially in a 3D modelling software environment is also multimodal. S-SL combines three theoretical constructs (i) *mental images* (Presmeg, 1986), (ii) *interpret figural information* (IFI) and *visual processing* (VP) (Bishop, 1983), and (iii) *Action*, *Production* and *Communication* space and the notion of *semiotic bundle* (Arzarello, 2008). These are used to look at the emergence of signs linked to spatial thinking. Following the multimodal paradigm, S-SL frames classroom productions that include specific signs, such as words, gestures, sketches and acts and so on, which are attachments to students', as well as the teacher's, spatial thinking processes. To do so, S-SL distinguishes spatial thinking as two major processes; IFI and VP. IFI includes the emergence of spatial vocabulary and the interpretation of visual images, while VP includes the emergence of Concrete Images (CI), Kinaesthetic Images (KI) and Dynamic Images (DI). CI can be considered as pictures in the visual memory, whereas KI refers to physical movements, and DI covers conceiving and manipulating dynamic mental images (Presmeg, 1986; Turgut, 2017). Figure 1 summarizes the S-SL and its components.

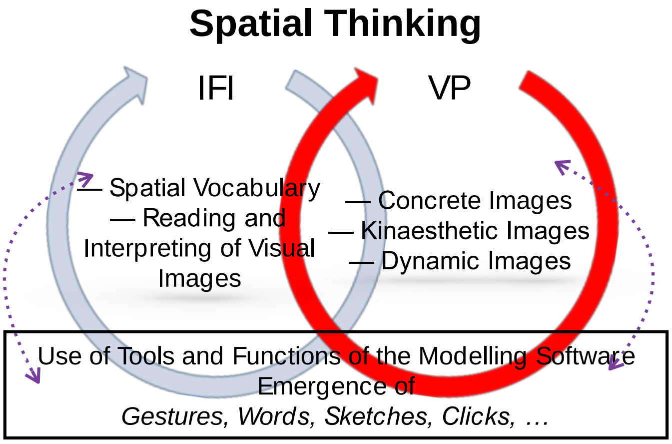


Fig. 1: S-SL with its components (modified from Turgut, 2017, p. 183)

Within the context of the present paper, we identify two strategies under IFI; a *spatial–analytic* strategy, meaning focusing on parts of the object, and a *spatial–holistic* strategy, which refers to comprehending and reasoning on the object as a whole. S-SL offers analysis on the emergence of signs through the notion of semiotic bundle (Arzarello, 2008), which constitutes two different, but complementary analysis tools; a *synchronic analysis* and a *diachronic analysis*. Synchronic analysis refers to 'the relationships among different semiotic resources simultaneously activated by the subjects at a certain moment', while a diachronic analysis means the 'evolution of signs activated by the subject in successive moments' (Arzarello, Paola, Robutti, & Sabena, 2009, p. 100).

METHODOLOGY

A task-based interview was conducted with an eighth grader, Atakan (pseudonym), who has a moderate level performance in mathematics. He has a desktop computer in his home and, as a result, he is competent in the use of basic computer tools. In order to research Atakan's spatial reasoning process, we considered 3D modelling software SketchUp® (SU) as an artefact, which is originally designed for engineering and model building. It should be noted that Atakan has experience in the use of SU since, as a part of a larger study he carried out 3D geometry tasks with the same software when he was in 7^{th} grade.

In the context of acquisitions described in the Turkish middle school mathematics curriculum, we prepared two interrelated but different tasks. During the interview, we first proposed three (top, front and right) views of a building (Figure 2a, 2b, 2c) made up of unit cubes and asked Atakan to construct the building. This initial task included two main steps; (i) *constructing the building* using concrete unit cubes provided and (ii) using virtual cubes within SU that provides a zero-gravity environment with the aim of *making alternative 3D buildings*. In the second task, we asked Atakan to complete 3D buildings within only the SU environment according to top and front views given on the paper (Figure 2d, 2e).

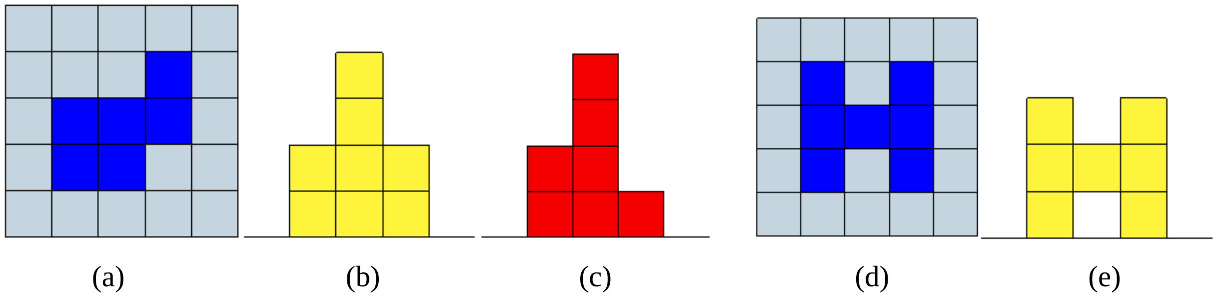


Fig. 2. (a) Top, (b) front, (c) right views in 1st task; (d) top, (e) front views in 2nd task

The video-recorded interview lasted about an hour. In order to capture signs, we used two cameras in different positions as well as screen recorder software. A thematic analysis (Braun & Clarke, 2006) was employed covering all the collected data to elaborate Atakan's reasoning steps.

SPATIAL-SEMIOTIC ANALYSIS OF THE DATA

For the sake of presenting an evolution of the student's reasoning, we first briefly present a macro analysis of the initial step of Task 1. As the first step, Atakan built the first floor of the building in a way to provide the top view (building blocks parallel to the ground) to form the structure with concrete cubes in accordance with the views given in the worksheet. In the second step, he built the cube block in a vertical position relative to the ground) to form the front view without changing the top view. In the third step, he compared the right view of the structure (with the right view given in the worksheet) changing the viewpoint by bending. Finally, in the fourth step, without changing the top and front views, he put a cube in an appropriate place to complete the right view. By the end of the process, Atakan had built a structure using twelve cubes.

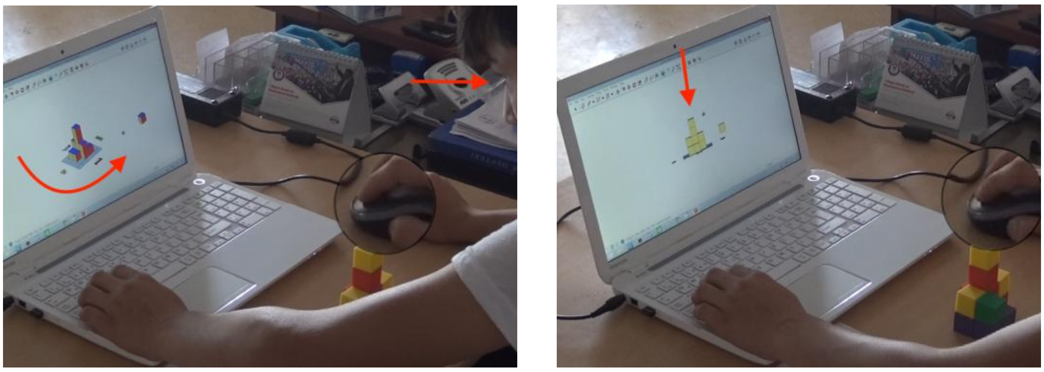
Synchronic and Diachronic Analyses of the Second Step of Task 1

Several spatial–semiotic resources appeared synchronously, when Atakan solved the second step of the Task 1 through SU. Table 1 briefly provides a summary of the most frequent spatial-semiotic resources categorized under the IFI and VP processes (SV: Spatial Vocabulary).

IFI		VP		
Spatial–Analytic	Spatial–Holistic	CI	KI	DI
–Exploring an appropriate viewpoint to add or remove a cube –Adding block cubes which are parallel or vertical to the base to obtain a top view –Focusing single views of the object	–Evaluating the object from different viewpoints –Reasoning which cubes that can be removed without changing the views –SV: expressing why front views isolated his strategies –SV: expressing strategies in relation to top and right views	–Using a mental picture derived from the paper and concrete object –Basing an obtained mental image in the completed (reasoning) step(s)	–Using the *Orbit* tool to complete different steps –Adding, moving or removing the cubes using the *Select* tool –Using the cursor for pointing out cubes or the object while explaining the situation	–Linking 2D and 3D representations mentally –Mental rotation with respect to given directions –Spatial orientation with respect to different viewpoints

Table 1. An overview of spatial–semiotic resources attached to the reasoning steps of Task 1

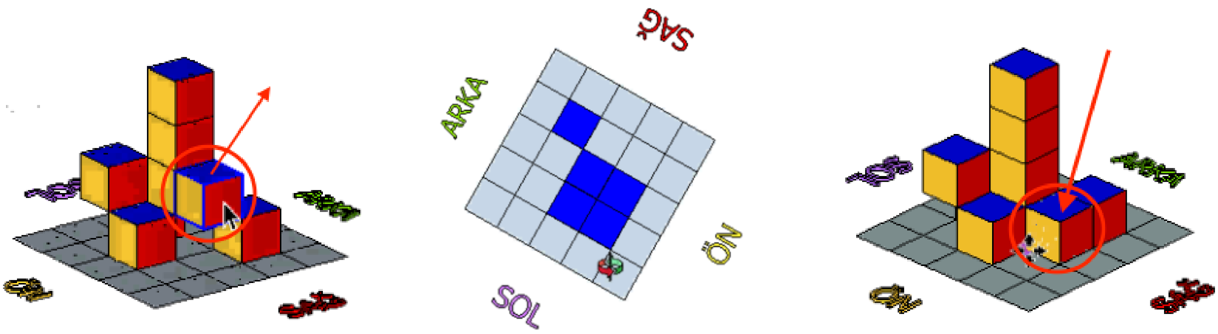
In order to present the emergence of specific resources expressed in Table 1, in the following statements, we summarize Atakan's reasoning steps for Task 1. At first, he repeated the steps in the initial part of Task 1 to create a representation of the structure (formed with twelve concrete unit cubes) in SU. In this process, by making use of the tool 'orbit', he made reasoning (using the tool slowly) about the procedures to be applied (KI, DI). He searched for a viewpoint appropriate to cube addition (using the tool fast) (KI) (Figure 3a), and he evaluated the top, front and right views of the structure he had formed (using the tool fast) (KI, CI) (Figure 3b). In the second part of Task 1, it was seen that without changing the top, front and right views, Atakan deleted a cube from the first floor in the process of transition from a 12-cube structure to an 11-cube structure (DI, KI), deleted a cube from the second floor in the process of transition to a 10-cube structure (DI, KI), deleted a cube from the first floor in the process of transition to a 9-cube structure (DI, KI), deleted a cube from the second floor in the process of transition to an 8-cube structure (DI, KI) and evaluated the views of the new structure at the end of each step (CI, KI).



(a) (b)

Fig. 3: (a) Atakan's exploration for a viewpoint (b) Evaluating the object from different viewpoints

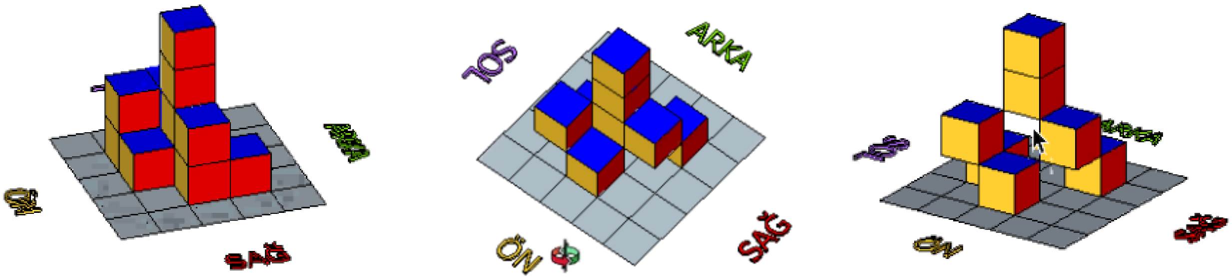
In the third part of Task 1, it was seen that without making any changes in the top and right views, Atakan changed the top view by deleting a wrong cube from the second floor in the process of transition from an 8-cube structure to a 7-cube structure (KI) (Figure 4a), recognized the wrong strategy in the second step (CI) (Figure 4b) and placed the cube (he had deleted) unintentionally in the first floor rather than in the second floor (Figure 4c) while trying to cancel this deletion (KI).



(a) (b) (c)

Fig. 4: (a) Deleting wrong cube, (b) Evaluating the top view, (c) Replacing the deleted cube

In the third step, Atakan examined the structure he had formed previously with concrete cubes when he failed to develop a strategy for transition from the 8-cube structure to the 7-cube structure, and he returned back to the 11-cube structure by adding cubes (KI) (Figure 5a). In the following steps, the participant used the cube-deletion strategy, respectively, to form 8-cube structure (Figure 5b), and finally to form the 7-cube structure (Figure 5c) that provided the top and right views and reached the correct result (DI, KI, CI). In the 2nd and 3rd parts of Task 1, Atakan, with the help of 'orbit'; (i) did reasoning in relation to solution strategies (using the tool slowly) (KI, DI), (ii) evaluated the views of the new structures formed (using the tool fast) (KI, CI) and (iii) searched for a viewpoint appropriate to cube-deletion and cube-addition (using the tool quickly) (KI). When the researchers asked Atakan why he had returned back to the 11-cube structure from the 8-cube structure, he replied, "*Well, it didn't work. I had formed according to the front view… the previous shape*" (SV).



(a) (b) (c)

Fig. 5: (a) 11-cube building, (b) 8-cube building, (c) 7-cube building

In order to summarize a combination of synchronic and diachronic analyses of Task 1, i.e., to articulate specific signs with respect to evolution of reasoning, we borrow the notion of *semiotic chain* in (Bartolini Bussi & Mariotti, 2008) and express Figure 6 to overview an evolution of Atakan's reasoning process.

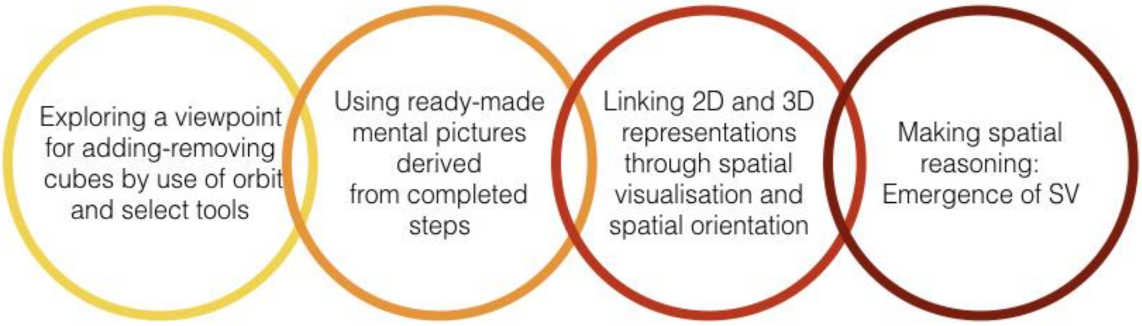
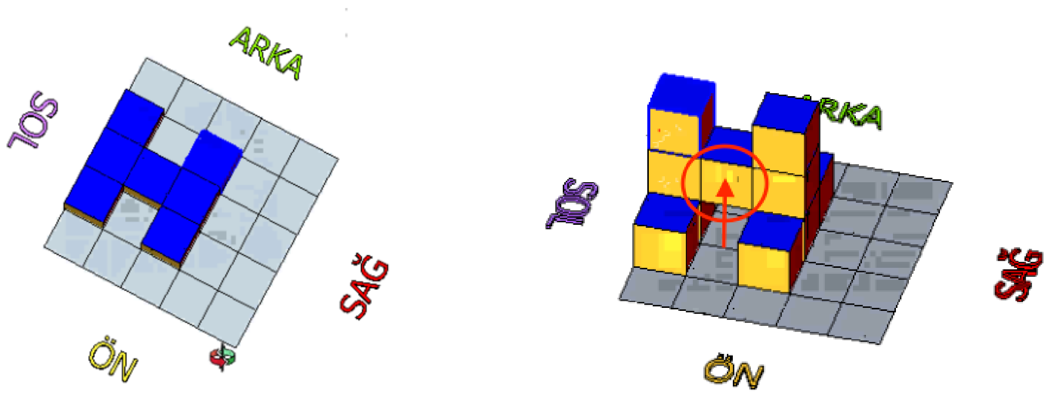


Fig. 6: A semiotic chain shows an evolution of Atakan's reasoning for Task 1

Synchronic and Diachronic Analyses of Task 2

Because the aims of the second step of the second task and the third task are close, the emergence of spatial-semiotic resources was similar to Table 1. However, in the second task, Atakan's strategies differed, where in this case two views of the building were provided on paper. Therefore, he exploited his experience coming from the first task and, in this way, he developed new insight for exploring the situation and all of this changed the IFI and VP columns in Table 1. Another fact is that, in the present case, the SV is more apparent compared to Task 2. Table 2 summarizes the emergence of specific signs.

Atakan first focused on building the first floor of the structure to form the top view in the first step and formed the cube-block in a position parallel to the ground. In this process, Atakan changed the viewpoint on the screen with the help of 'orbit' (fast use) to add the cubes where needed (KI). In addition, the participant considered the direction codes on the screen and rotated the image for the top view in his mind as appropriate to the direction codes while building the first floor of the structure (DI) (Figure 7a).



(a) (b)

Fig. 7: (a) Initial structure, (b) Moving the cube from the first to second floor

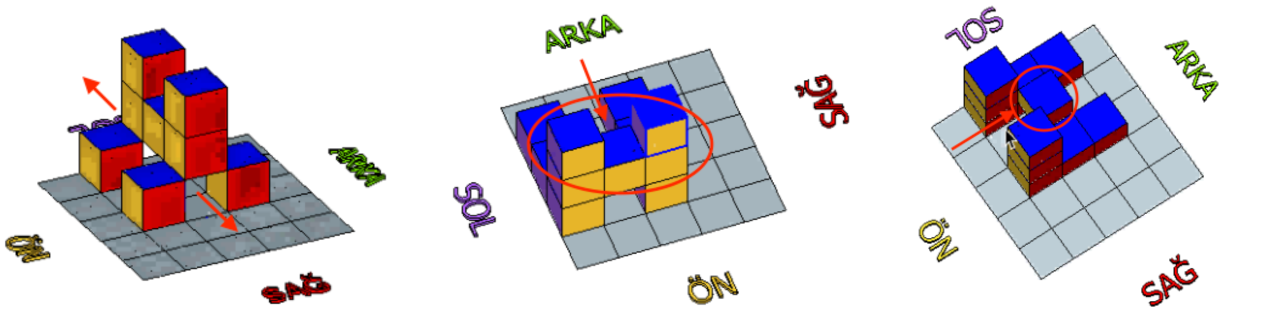
In the second step, Atakan focused on the second and third floors of the structure as appropriate to the front view given in the worksheet and realized that the first floor he had formed in the first step provided the top view but not the front view (CI). In addition, he carried one of the cubes from the first floor to the second floor (DI, KI) (Figure 7b).

IFI		VP		
Spatial-Analytic	Spatial-Holistic	CI	KI	DI
–Exploring an appropriate viewpoint to add or remove a cube –Adding block cubes which are parallel or vertical to the base to obtain a top view –Focusing single views of the object –SV: emphasis on a partial solution strategy –Work (temporarily) on the cubes that satisfy a front view while not satisfying a top view –SV: evaluating the different views part by part –SV: explaining why he could not develop a strategy for removing cubes with respect to the floors of the cubes –Focusing only on removal of the cubes, and as a result of this, failing to visualise of the object with 7 cubes	–Evaluating the object from different viewpoints –Reasoning on the cubes that can be removed but which do not change the views –SV: explaining and pointing out the cubes that can be moved but also satisfying the views –Determining symmetric cubes satisfying two different views when they are deleted –SV: reasoning on the relationship between the front and rear views – Building the object from the beginning for developing new strategies –SV: a new strategy for moving cubes on the third floor –SV: generalizing strategy of removing cubes to satisfy top view	–Using a mental picture derived from the paper –Basing single views (top, front and so on) of the object – Basing obtained mental images in the completed (reasoning) step(s)	–Using the ‘*Orbit*’ tool to complete different steps –Adding, moving or removing the (symmetric) cubes and/or *blocks* using the ‘*Select*’ tool –Using the cursor for pointing out cubes or the object while explaining the situation –Using the zoom in-zoom out tool –Deleting the whole object	–Linking 2D and 3D representations mentally –Mental rotation with respect to given directions –Spatial orientation with respect to different viewpoints –Visualising new views of the object when some cubes are moved –Visualising different views synchronously in the case of removing and/or moving the cubes

Table 2. A summary of spatial–semiotic resources attached to reasoning steps of Task 2

Following this process, Atakan built a cube block in a vertical position to the ground (KI) and formed a structure that provided the front view. In addition, it was seen that Atakan searched for a viewpoint appropriate to cube addition with the help of ‘orbit’ (fast use) (KI) and evaluated views of the structure (CI). In the second part of Task 2, Atakan focused on building the structures that provided top and front views using fewer cubes. In this process, Atakan focused on symmetrical cube pairs on the right and left sides that did not change the top and front views when deleted (DI) and he deleted two symmetrical cubes from the first floor (KI) (Figure 8a). Following this, while the participant evaluated the top and front views of the new structure with the help of 'orbit' (fast use) (CI, KI), the researchers asked him whether there was an alternative solution, which included nine cubes. Within the scope of this question, it was seen that Atakan initially replaced again the two symmetrical cubes he had deleted (KI) and then simultaneously examined the 11-cube structure and the views given in the worksheet to produce new strategies (DI). In such a way, it was also seen that Atakan examined the structure from different viewpoints with the help of 'orbit' (slow use) (KI), searched for the cubes that would not change the views when deleted, and failed to produce solution strategies.

Therefore, the researchers asked Atakan whether he would be able to form an alternative 11-cube structure with the same top and front views. Within the scope of this question, to begin with, Atakan simultaneously examined the 11-cube structure and the views given in the worksheet (DI) and then said the block which formed the second and third floors could be moved one unit backward or one unit forward (DI, SV). In the following step, he moved this cube block one unit forward (KI) (Figure 8b). Following this, Atakan, with the help of 'orbit' (fast use), evaluated the top and front views of the structure (CI, KI) and saw that the top view had changed. As a result, he moved one cube on the second floor to provide the top view (KI) (Figure 8c). Following this step, Atakan evaluated the views with the help of 'orbit' (CI, KI), realized that the top view was again wrong and deleted one cube in the second floor, which changed the top view (KI). Following this strategy, in which the participant did not change the top and front views, he evaluated the views with the help of "orbit" again (fast use) (KI, CI) and said that the alternative 11-cube structure was complete (SV).



(a) (b) (c)

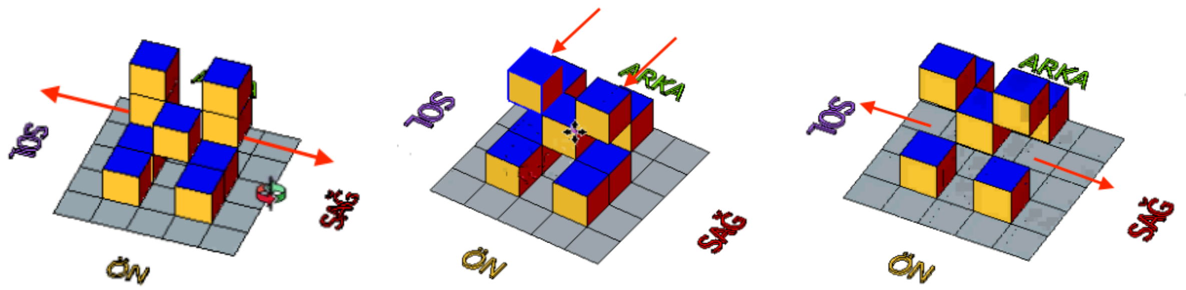
Fig. 8: (a) Deleting symmetrical cubes, (b) Moving cube block forward, (c) Moving the cube backward

In the next part, the researchers asked Atakan whether he could work on the structure and form an alternative 9-cube structure. Within the scope of this question, the participant, with the help of 'orbit' (slow use), searched for symmetrical cubes, which would not change the top and front views when deleted (KI, CI, DI) and said that he failed to find a strategy to form such a structure (SV). In the following process, Atakan continued his search with 'orbit' (slow use) (KI) and realized that there would be no change in the top and front views when two symmetrical cubes in the first floor were deleted. The participant deleted the symmetrical cubes he had determined (KI), and he then evaluated the top and front views of the new structure with the help of 'orbit' (fast use) (CI, KI).

In the final part, the researchers asked Atakan whether he could form a 7-cube structure without changing the top and front views. Within the scope of this question the participant, with the help of 'orbit' (slow use), searched for cubes that would not change the top and front views when deleted (KI, CI, DI). As a result, Atakan reasoned in relation to the 9-cube structure and the views in the worksheet (DI), but failed to develop a strategy to form the 7-cube structure at the end of the process. Eventually, he deleted all the cubes on the screen to re-form the 11-cube structure (KI, SV).

Atakan, who started building the structure again, this time formed the cube block in a vertical position to the ground to complete the front view (CI, KI). This block was built in such a way as to form the rear of the structure differently from his previous structures.

In the next part, the participant examined the structure from the top with the help of 'orbit' (fast use) (KI) and saw that one of the cubes he had added to the second floor changed the top view (CI). Therefore, he moved this cube one unit forward (DI, KI). Following this, Atakan worked on the cube block in a position parallel to the ground and built the first floor (KI) in such a way as to complete the top view without changing the front view (CI, DI). As a result, he completed the 11-cube structure. The participant deleted two symmetrical cubes from the first floor during transition to the 9-cube structure (KI) (Figure 9a). Next, he used the 'zoom' tool to examine the structure in more detail (KI). Lastly, with the help of 'orbit' (slow use), he searched for cubes he could delete to make transition to the 7-cube structure (KI). In this process, Atakan did reasoning in relation to the 9-cube structure and regarding views given in the worksheet (CI, DI). He said that he did not make transition to the structure with the cube-deletion strategy as demanded in the question (DI, SV). In this respect, when the researchers asked Atakan whether he had developed his thinking strategy based on a cube-deletion strategy, he responded positively to this question and said he would think about the structure a bit more and move two symmetrical cubes in the third floor one unit forward. He then added that these cubes would hang in the air at the end of the process without changing the views (DI, SV). Following this, the participant moved the symmetrical cubes in the third floor one unit forward (KI) (Figure 9b). Next, he added the cubes to places where he wanted and examined the structure with the tool of 'zoom' (KI). After this, he deleted the symmetrical cubes in the first floor, which were under the symmetrical cubes he had moved forward (DI, KI) (Figure 9c). In the last step, Atakan evaluated the top and front views with the help of 'orbit' (fast use) (CI, KI).



(a) (b) (c)

Fig. 9: a, b, c Process of transition from 9-cubes building to 7-cubes building

At the end of the solution process, the researchers asked Atakan to explain his reasoning processes, and he said that the cubes placed under one cube in the upper floors were not visible from the top view and that deleting the cubes below would not change the top view (SV). In this respect, Atakan reported, "*from the top view, we see the upper cubes, and the ones below are not visible. If we take the ones below, those at the top look the same*". In addition, Atakan stated that he evaluated how simultaneously the deletion process, which did not change the top view, did not change the front view (SV). In this respect, Atakan said, "*When we did not move to the front and if I take these* (showing the symmetrical cubes he had deleted from the first floor in the last step), *then these* (coming to the top view rapidly with the help of 'orbit') *would have looked as if they had been removed* (pointing to the procedure that changed the top view). *In addition, if I had taken these* (showing the symmetrical cubes in the second floor at the back) ... *they would have remained at the back* (showing the symmetrical cubes in the third floor he had moved one unit forward) … *Then they would have looked* ... (taking the front view rapidly with the help of 'orbit' and showing the spaces that would appear in the front view at the end of the process)."

Figure 10 refers to a combination of synchronic and diachronic analyses of Atakan's reasoning processes associated with Task 2.

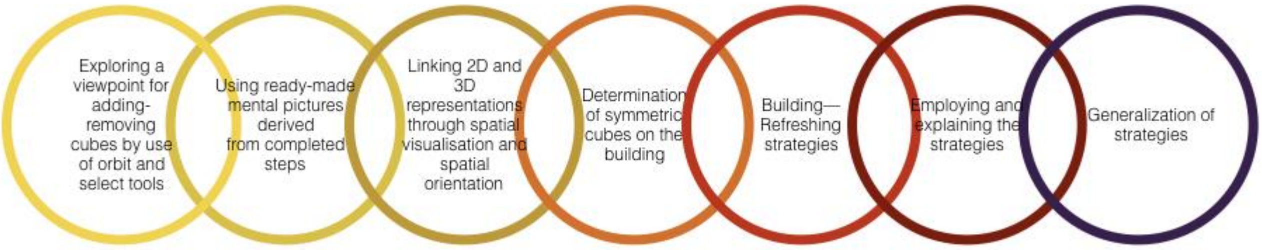


Fig. 10: A semiotic chain shows an evolution of Atakan's reasoning for Task 2

CONCLUSIONS

In this paper, we consider the following research question: 'What kind of spatial-semiotic resources emerge when an eighth-grade student solves spatial tasks using 3D modelling software?' Spatial-semiotic analyses of the data obtained provided us with a detailed understanding of student's spatial reasoning processes in SU. In the first task, the student easily built the structure with concrete unit cubes whose different views provided on the paper. In the second task, the student's reasoning steps appeared with an emphasis on a spatial-analytic strategy based on exploring a viewpoint for adding or removing cubes, using ready-made mental pictures, linking 2D and 3D representations through spatial visualisation and spatial orientation, and an emergence of spatial vocabulary, including his strategies. However, in the third task, certain specific reasoning steps appeared as spatial-holistic strategies more than in the previous task, such that focusing on an environment with zero-gravity, symmetric cubes, and constructing and explaining strategies, interlaced into completed steps in the second task. Within the context of our study, gestures were limited to in the use of specific tools ('orbit', 'select', mimicking with cursor, ctrl+v, delete and 'zoom'). There did not appear to be any gestures independent of the artefact (mouse and keyboard), such as hand movements, tracing with a finger and so on.

In terms of the obtained results, we finally summarize the synergies among the KI, CI, DI and VP and IFI processes through Figure 11, which are a theoretical contribution and an attachment to Figure 1.

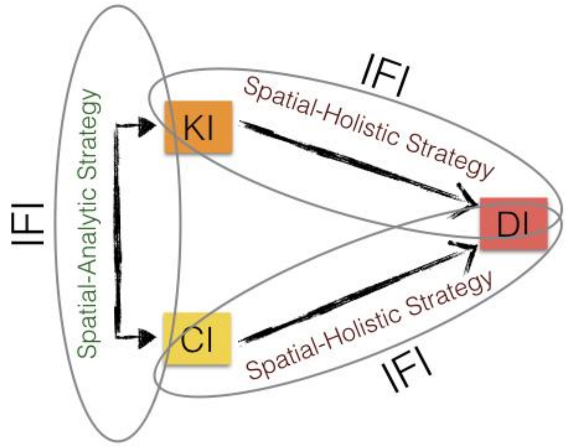


Fig. 11: Synergies between spatial thinking processes

Figure 11 implies that spatial–analytic and spatial–holistic strategies that we consider in this paper commonly intertwined with IFI process and emergence of KI, CI and DI. IFI process always emerged when the student solved spatial tasks and this appears to be that IFI is *the core element* in spatial thinking and creation of DI. The emergence of signs confirmed that the student's initial strategy was spatial–analytic, and specific images were KI and CI. The next step was emergence of DI in terms of spatial–holistic strategy and IFI process. However, these results come from only an eighth grader's result, it will be meaningful to explore a group of students' results to discuss articulation of Figure 1 and Figure 11 in a further research.

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