# DESIGNING TASKS THAT FOSTER MATHEMATICALLY BASED EXPLANATIONS IN A DYNAMIC SOFTWARE ENVIRONMENT

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*This poster introduces the main ideas behind a study conducted during the spring 2017. The aim was to investigate how different formulations of tasks, where students are expected to provide mathematically based explanations, might influence their responses. Preliminary results from the first stage in the analysis process indicate that there are some interesting differences due to small differences in task formulation.*

*Keywords: dynamic mathematics software, task design, mathematically based explanation*

## BACKGROUND

The increased availability of different kinds of technology in mathematics classrooms offers new possibilities, but it requires change in teaching and learning practice. For example there is a need for different kinds of task to utilize the affordances provided by new technology (Hegedus et al., 2017). Recently, the issue of designing tasks suitable in the digital mathematics classroom had an entire book devoted to it: *Digital Technologies in Designing Mathematics Education Tasks* edited by Leung and Baccaglini-Frank (2017). The literature suggests various task design principles to promote mathematical reasoning in a Dynamic Mathematics Software (DMS) environment, the particular technology used in this study.

This study builds on our previous work on developing new types of task environment to foster students' mathematical reasoning (Brunström & Fahlgren, 2015; Fahlgren & Brunström, 2014). One result from a design-based research project, conducted in a DMS environment by the authors of this paper in collaboration with four upper-secondary school teachers, showed that students' explanations tended to be superficial and more descriptive than explanatory. These results are in line with results from other studies showing that there is a risk that students do not reflect on the mathematics involved when using DMS to explore and conjecture (e.g. Drijvers, 2003; Healy & Hoyles, 1999; Joubert, 2013).

It is important for task designer to be aware that small differences in the formulation of tasks might have significant impact on students' responses (Sierpinska, 2004). We found that the wording is crucial in the formulation of questions where students are asked for explanations (Brunström & Fahlgren, 2015). So far, however, there are few studies that have investigated how small changes in wording might influence students' explanatory responses in a DMS environment.

This study compares two different ways of formulating explanation tasks in a DMS environment. The explanation tasks are embedded in a task sequence with the aim of developing students' awareness of some of the connections between the standard form of quadratic function $f(x) = ax^2 + bx + c$ and the corresponding graphical representation and quadratic equation. In total, the task sequence includes three explanation tasks formulated in the following two versions: (A) “Explain why…” and (B) “Give a mathematical explanation why…”. The aim with the study is to investigate if this small difference in task formulation has any impact on student responses.

## THE STUDY

The study involves seven $10^{th}$ grade upper-secondary classes in which half of each class received the A-version and the other half received the B-version of the task sequence. The students worked in pairs with *one* computer per pair. The purpose of this is that the computer screen should provide a shared object for discussions between students (Brunström & Fahlgren, 2015; Paiva, Amado, & Carreira, 2015). The empirical data consists of the written responses from 229 students; 121 version A, and 108 version B.

## PRELIMINARY RESULTS

So far, a preliminary analysis of the first explanation task, with focus on parameter $c$, has been made. In this task, the students are asked: (1) to investigate and find out how the value of $c$ affects the graph; then (2) how the value of $c$ can be found in the coordinate system; and finally (3) to (A) Explain why/(B) Give a mathematical explanation why the value of $c$ can be found in this way. When answering the second subtask, almost all students described that the value of $c$ can be found where the graph intersects the $y$-axis. Our focus in the analysis was on student responses on the third subtask. The tables below indicate differences in student responses, both in terms of types of explanation (Table 1) and forms of representation (Table 2).

| **TYPE OF EXPLANATION** | **VERSION A** | **VERSION B** |
|---|---|---|
| Correct and complete, i.e. explains that $c = f(0)$. | 4 % | 13 % |
| Refers to the $b$-value in the straight line equation $y = mx + b$ | 44 % | 57 % |
| Describes that $c$ can be found where the graph intersects the $y$-axis (i.e. repeats the answer to the previous subtask) | 21 % | 13 % |
| Provides more than one explanation. | 16 % | 29 % |

**Table 1. Some differences in student responses concerning types of explanation**

The preliminary results indicate that the B-version that includes the words “mathematical explanation” prompts student responses based on mathematical properties and relations to a higher degree than the A-version does. Even if not many students gave a correct and complete explanation this was more frequent among students responding to the B-version. We also found it interesting that these students more often referred to their previous knowledge concerning the straight line equation, and also gave more than one explanation to a greater extent.

| FORM OF REPRESENTATION | VERSION A | VERSION B |
|---|---|---|
| Verbal only | 75 % | 42 % |
| Algebraic Symbols only | 1 % | 16 % |
| Verbal and Algebraic Symbols | 4 % | 17 % |
| Verbal with Elements of Algebraic Symbols | 11 % | 18 % |
| No answer | 9 % | 7 % |

**Table 2. Some differences in student responses concerning types of representation**

The preliminary results also indicate that the task formulation including "mathematical" prompts more students to use algebraic symbols in their explanations, and fewer to use solely verbal explanations. In student responses classified as "Verbal with Elements of Algebraic Symbols" formulas or other algebraic symbols are just mentioned without being used. Hence, the categories "Algebraic Symbols only" and "Verbal and Algebraic Symbols" are the only categories where students really use algebraic symbols (even if not always in an appropriate way). When merging these two categories the tendency becomes clear, 33 % of the students answering the B-version used algebraic symbols while the corresponding value for those responding to the A-version was 5 %.

The next step in the analysis process, is to develop a more general framework to use in the analysis of all three explanation tasks.

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